

國立中央大學八十五學年度碩士班研究生入學試題卷

所別: 大氣物理研究所 不分組 科目: 應用數學 共 / 頁 第 / 頁

1. Find a general solution of the following equation

$$x^2 y'' - 4xy' + 6y = 1/x^4 \quad (15\%)$$

2. Find the eigenvalues and eigenfunctions of the following problem.

$$x^2 y'' + xy' + (\lambda^2 x^2 - 1)y = 0, \quad y(0) = 0, \quad y(1) = 0. \quad (15\%)$$

3. Find a basis of eigenvectors and diagonalize

$$\begin{bmatrix} 5 & 0 & -6 \\ 2 & 1 & -4 \\ 3 & 0 & -4 \end{bmatrix} \quad (15\%)$$

4. Using Laplace transforms, solve the following initial value problem.

$$\begin{aligned} \frac{d^2 x}{dt^2} + y &= -5 \cos 2t & x(0) &= 1, & \frac{dx}{dt}(0) &= 1; \\ \frac{d^2 y}{dt^2} + x &= 5 \cos 2t & y(0) &= -1, & \frac{dy}{dt}(0) &= 1. \end{aligned} \quad (15\%)$$

5. Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n} dA$ by the divergence theorem, where

$$\mathbf{F} = x^3 z^2 \mathbf{i} + y^3 z^2 \mathbf{j} + x^2 z \mathbf{k}, \text{ and } S \text{ is the surface of } x^2 + y^2 \leq a^2, |z| \leq h \quad (15\%)$$

6. Find

$$\int_0^{\infty} \frac{\cos xw}{1+w} dx, \quad \text{if } x > 0. \quad (10\%)$$

7. Evaluate

$$\oint_C \frac{e^{-z^2}}{\sin 4z} dz, \quad \text{where } C \text{ is the unit circle (counterclockwise)} \quad (15\%)$$