

1. 一半徑為 1 的圓球，球心位於原點。 $\vec{F}$  代表空間中的質量通量場，其表示方式為

$$\vec{F} = 3x\vec{i} + (e^y + x^2)\vec{j} + (\ln y + z)\vec{k} \quad \left( \frac{\text{kg}}{\text{sec} \cdot \text{m}^2} \right)$$

請問每秒流入(出)這個球的質量為多少 kg?

(15%)

2.  $\vec{V}$  代表一個二維速度場， $\vec{V} = x^2\vec{i} + y\vec{j}$ 。求出通過點 $(x_0, y_0)$ 的流線(streamline)方程。

(15%)

3. 在二維的情況下，且 $\frac{\partial}{\partial z} = 0$ ，一向量 $\vec{V}$ 由無輻散(nondivergent)部分和無旋轉(irrotational)部分組成，即

$$\vec{V} = \vec{V}_1 + \vec{V}_2, \text{ 而 } \vec{V}_1 = -\nabla\phi, \vec{V}_2 = \vec{k} \times \nabla\psi \quad \text{其中} \left( \nabla_1 = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} \right)$$

(i). 請問 $\vec{V}_1, \vec{V}_2$ 何者為無輻散? 何者為無旋轉?

(ii).  $\nabla \cdot \vec{V}$  和  $\nabla \times \vec{V}$  各為何?

(15%)

4. 解釋下列名詞並說明其物理意義：

(a). Reynolds number ; (b). Froude number ; (c). Euler number .

(10%)

5. Drag coefficient depends on what dimensionless parameters? Explain how these parameters affect drag coefficient. What factors determine lift? What is the most important parameter that affects the lift coefficient? Why the ratio of the lift to drag is important for lift-generating devices?

(15%)

6. Write down the Navier-Stokes equations. State the meaning of each term. What is Bernoulli equation? Under what conditions it can be derived from the Navier-Stokes equation?

(15%)

7. Assume that an incompressible flow has  $U$  as the characteristic velocity scale,  $L$  the characteristic length scale,  $L/U$  the characteristic time scale,  $\Delta p$  the characteristic pressure difference scale,  $g$  the gravity,  $\rho$  the density and  $\mu$  the viscosity coefficient. Using above characteristic scales to nondimensionalize the Navier-Stokes equation. State the meaning of each non-dimensional parameter.

(15%)