國立中央大學九十一學年度碩士班研究生入學試題卷

所別： 大氣物理研究所 不分組 科目： 應用數學 共 2 頁 第 1 頁

1. The heat conduction equation can be written as:

$$\nabla^2\theta = \frac{C \cdot \rho_v}{\kappa}\frac{\partial\theta}{\partial t}$$

where θ is temperature, t is time, and C, P_v, κ are constants. If S is a closed surface over which the temperature θ is zero. Please show that:

$$\iiint_V C \cdot \rho_v \cdot \theta \cdot \frac{\partial\theta}{\partial t} dV = -\iiint_V \kappa \,|\nabla\theta|^2 dV$$

Note that V is the volume enclosed by S

(10%)

2. (a). For a curvilinear coordinate system (q_1, q_2, q_3), the so-called scale factor h_i can be obtained by:

$$h_i^2 = \sum_{j=1}^{3}\left(\frac{\partial x_j}{\partial q_i}\right)^2 \qquad \text{where } i = 1\sim 3$$

Note that (x_1, x_2, x_3) correspond to the Cartesian coordinates (x, y, z).
Please compute the scale factors for a cylindrical coordinate system (r, θ, z), where

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$z = z$$

(b). The curl of a vector $\vec{F} = F_1\vec{u}_1 + F_2\vec{u}_2 + F_3\vec{u}_3$, represented by the curvilinear coordinate system, can be denoted by:

$$\nabla \times \vec{F} = \frac{1}{h_1 \cdot h_2 \cdot h_3}\begin{vmatrix} h_1\vec{u}_1 & h_2\vec{u}_2 & h_3\vec{u}_3 \\ \dfrac{\partial}{\partial q_1} & \dfrac{\partial}{\partial q_2} & \dfrac{\partial}{\partial q_3} \\ F_1h_1 & F_2h_2 & F_3h_3 \end{vmatrix}$$

where $\vec{u}_i$ is a unit vector in the direction of increasing q_i.
Suppose a wind field is specified on a cylindrical coordinate by:

$$\vec{V} = V_r\vec{u}_r + V_\theta\vec{u}_\theta$$

in which

$$Vr = \left(\frac{r}{r_{max}}\right)^{\lambda_1} \qquad V_\theta = \left(\frac{r}{r_{max}}\right)^{\lambda_2}$$

r_{max} is a pre-determined radius, and λ_1, λ_2 are two constants. Please compute $\nabla\times\vec{V}$ for this wind field.

(c). What is $\nabla\times\vec{V}$ at r=0.

(20%)

3. Solve the following problem

(a). $x^2y'' + xy' + (\lambda^2x^2 - 1)y = 0$, $\quad y(0) = y(1) = 0$

(b). $y'' - 4y' + 3y = 10\sin x$, $\quad y(0) = 2$, $y'(0) = 1$

(20%)

參考用

注意：背面有試題

4. Using Laplace transforms, solve the following integral equation

$$y(t) = te^{t} - 2e^{t}\int_0^t e^{-\tau} y(\tau)\,d\tau$$

(10%)

5. Find out what type of conic section is represented by the following quadratic form. Transform it to principal axes. Express $\mathbf{x}^T = [x_1 \quad x_2]$ in terms of the new coordinate vector $\mathbf{y}^T = [y_1 \quad y_2]$

$$7x_1^2 + 6x_1x_2 + 7x_2^2 = 200$$

(15%)

6. Find the Fourier series of the following periodic function

$$f(x) = |\cos x|, \qquad -\pi < x < \pi$$

(10%)

7. Solve the following partial differential equation

$$u_{xx} + 6u_{xy} + 9u_{yy} = 0$$

(15%)