系所別：　　　大氣物理研究所　　　科目：　　　　　應用數學

1. If $\vec{F} = \nabla \times \vec{A}$, prove that the integration of $\vec{F}$ over any closed surface is zero.
(10%)

2. Let $\vec{F} = y^2\,\vec{i} + 2xy\,\vec{j}$. If $\vec{F}$ is integrated along curve $y = x^2$ from (1,1) to (2, 4), please compute the result of this line integration.
(10%)

3. Explain the following terms.
   a. Linear and nonlinear differential equations;
   b. Exact differential equation;
   c. Superposition principle;
   d. Free oscillation and forced oscillation;
   e. Resonance
(10%)

4. Show that the Constant-Coefficient Equations and the Euler-Cauchy Equations can be transformed into each other.
(10%)

5. Write down the following formula.
   a. Definition of Laplace transform
   b. First Shifting theorem (s-Shifting)
   c. Differentiation of function
   d. Differentiation of transform
   e. Convolution
(10%)

6. What is the method of separating variables in solving partial differential equations? Give an example. In separating the heat equation we got exponential functions. Why? Why not in the case of the wave equation?
(10%)

7. Why is orthogonality of function important? How is it defined? What is a Sturm-Liouville problem? What does it have to do with orthogonality?
(10%)

8. Find a basis of eigenvectors and diagonalize the following matrix

$$\begin{bmatrix} -8 & 11 & 3 \\ 4 & -1 & 3 \\ -4 & 10 & 6 \end{bmatrix}$$
(15%)

9. Find the Fourier transform of the following function,

$$f(x) = \begin{cases} e^x & \text{if } -a < x < a \\ 0 & \text{otherwise} \end{cases}$$
(15%)