所別：大氣物理研究所碩士班 一般生 科目：流體力學

1. Show that

$$\frac{\partial \rho \vec{\mathbf{V}}}{\partial t} + \nabla \cdot \rho \vec{\mathbf{V}} \vec{\mathbf{V}} = \frac{\partial \rho \vec{\mathbf{V}}}{\partial t} + (\vec{\mathbf{V}} \cdot \nabla)\vec{\mathbf{V}} + \vec{\mathbf{V}}(\nabla \cdot \rho \vec{\mathbf{V}}) = \rho \frac{d\vec{\mathbf{V}}}{dt}$$

where ρ is fluid density and $\vec{\mathbf{V}}$ wind vector. (10%)

2. A mathematical formula for the curl of the wind vector $\vec{\mathbf{V}}$ is given by

$$\nabla \times \vec{\mathbf{V}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \frac{1}{r} \begin{vmatrix} \hat{\mathbf{e}}_r & r\hat{\mathbf{e}}_\theta & \hat{\mathbf{e}}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ v_r & rv_\theta & v_z \end{vmatrix}$$

in Cartesian coordinates and cylindrical coordinates. A cylinder fluid purely rotates about its vertical axis with a constant angular velocity $\Omega\,\hat{\mathbf{e}}_z$, which thus forms a forced vortex. Here, u, v and w are the three orthogonal components of the wind $\vec{\mathbf{V}}$ in Cartesian coordinates, and v_r, v_θ and v_z in cylindrical coordinates.

(a) Show that the associated vorticity vector ζ is constant and is equal to $2\Omega\,\hat{\mathbf{e}}_z$. (5%)

(b) Derive $u = v_r \cos\theta - v_\theta \sin\theta$ and $v = v_r \sin\theta + v_\theta \cos\theta$. (5%)

(c) Is the flow nondivergent? Why? (5%)

(d) Is the flow deformational? Why? (5%)

3. The Reynolds transport theorem relates changes between the system (SYS) and control volume (CV) such that $\frac{D}{Dt}\iiint_{SYS} \rho b\, d\mathcal{V} = \frac{\partial}{\partial t}\iiint_{CV} \rho b\, d\mathcal{V} + \oiint_{CS} \rho b \vec{\mathbf{V}} \cdot \hat{\mathbf{n}}\, dA$ where CS is the surface of the control volume with unit vector $\hat{\mathbf{n}}$ normal to the differential surface dA and velocity $\vec{\mathbf{V}}$ on dA, and b is some physical property of the fluid.

(a) Derive the mass-conservation equation from this theorem. (5%)

(b) Derive the momentum equation from this theorem. (5%)

(c) Use Gauss's divergence theorem to derive the mass-conservation equation in differential form. (5%)

4. Describe the streamline coordinates and the associated flow accelerations along and normal to the streamline? (10%)

5. (a) Show that a plane free vortex ($v_\theta = K/r$ with constant positive K) is irrotational. (5 %)

(b) Determine the circulation of the free vortex for *any path* enclosing the origin (r =0). (5%)

(c) Determine the circulation of the free vortex for *any path* excluding the origin (r =0). (5%)

(d) Derive streamfunction ψ and velocity potential ϕ, *and plot the flow net.* (10%)

6. Please write down the Navier-Stokes momentum equations for incompressible Newtonian flow and express your idea how to nondimensionalize the equations. (10%)

7. Describe the purpose of the Buckingham Pi theorem for dimensional analysis, and explain how you choose repeating variables and why the repeating variables cannot form a dimensionless product. (10%)