1. Diagonalization of the matrix $A = \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix}$.

2. Transform $Q = x_1^2 + 6x_1x_2 + 9x_2^2 = 10$ to the principal (new) axes and determine the new axes using the old axes $(x_1, x_2)$.

3. (a) Find the Fourier integral representation of the function
$$f(x) = \begin{cases} k & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$
(b) From (a), show that
$$\int_0^{\pi} \frac{\cos x \sin x}{x} \, dx = \frac{\pi}{4}$$

4. Solve the heat equation in a finite bar of length $L$
$$u_t = c^2 u_{xx} \quad (0 \leq x \leq L, t > 0)$$
with the following boundary conditions
$$u_x(0, t) = 0 \text{ and } u_x(L, t) = 0,$$
and the initial condition
$$u(x, 0) = f(x),$$
where $c$ and $k$ are real constants.

5. Solve the following Bernoulli differential equation.
$$y' + 2y = y^2$$

6. Solve the following initial value problem,
$$y_1' + \alpha y_1 = -\alpha (y_1 - y_2)$$
$$y_2' + \alpha y_2 = \alpha (y_1 - y_2)$$
And $y_1(0) = a_1$, $y_1'(0) = a_1$, $y_2(0) = b_1$, and $y_2'(0) = b_1$.

7. Find the eigenvalues and eigenfunctions of the following problem
$$(xy')' + \lambda x^{-1} y = 0, \quad y(1) = 0, \quad y'(e) = 0$$

8. Find the inverse Laplace transform of the following function
$$\frac{s^5 + 3(s+1)^3}{s^5(s+1)^3}$$