

國立中央大學98學年度碩士班考試入學試題卷

所別：大氣物理研究所碩士班 一般生 科目：應用數學 共 / 頁 第 / 頁

*請在試卷答案卷(卡)內作答

1. Solve the following initial value problem.

$$y'' + 4y = 8e^{-2x} + 4x^2 + 2, \quad y(0) = 2, \quad y'(0) = 2$$

(15%)

2. Show that the two large classes of differential equations, i.e., Constant-Coefficient Equations and Euler-Cauchy Equations, can be transformed into each other.

(15%)

3. What is the main difference between a power series method and a Frobenius method? What is a Sturm-Liouville problem? Why it is so important? What is orthogonality of functions? What is an orthogonal eigenfunction expansion?

(15%)

4. Write down the following formula of Laplace transform.

- The definition of Laplace transform;
- The first and the second Shifting theorems;
- Laplace transforms of the derivative and integral of a function;
- The function corresponds to the derivative and integral of a Laplace transform;
- Convolution.

(15%)

5. Diagonalize the following matrix.

$$\begin{bmatrix} -8 & 11 & 3 \\ 4 & -1 & 3 \\ -4 & 10 & 6 \end{bmatrix}$$

(15%)

6. What is Green's theorem? What is divergence theorem? What is Stokes's Theorem? When and where you can apply these theorems? Show some examples.

(10%)

7. Solve the two-dimensional wave equation,

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right),$$

that satisfies the following boundary and initial conditions,

$$u(0, y, t) = u(a, y, t) = u(x, 0, t) = u(x, b, t) = 0$$

and

$$u(x, y, 0) = f(x, y), \quad \frac{\partial u}{\partial t} \Big|_{t=0} = g(x, y)$$

(15%)

參考用