國立中央大學101學年度碩士班考試入學試題卷

所別：地球科學學系地球物理碩士班 不分組(一般生)　科目：微積分　共 2 頁 第 1 頁
地球科學學系地球物理碩士班 不分組(在職生)

本科考試禁用計算器　　＊請在試卷答案卷（卡）內作答

作答時須列出完整計算過程

1. (a) $\lim_{x\to a}\frac{x^x-a^a}{a^x-x^a}=?$ [5%]

(b) $\lim_{x\to 0}(\csc x-\cot x)=?$ [5%]

2. (a) $\frac{d}{dx}\ln\left(\frac{x-1}{x+1}\right)=?$ [5%]

(b) $\frac{d}{dx}\left(\tan^{-1}\frac{y}{x}\right)=?$ [5%]

3. (a) $\int e^{-2x}\cos nx\,dx=?$ [5%]

(b) $\int_{-l}^{l}\left(\cos\frac{m\pi x}{l}\right)\left(\cos\frac{n\pi x}{l}\right)dx=?$ (m, n 為整數) [5%]

4. Solve the initial value problem $x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=0,\ y(1)=1,\ y'(1)=0.$ [10%]

5. Find the eigenvalues and eigenvectors of the matrix. [10%]

$$A=\begin{bmatrix}5 & -2\\ 9 & -6\end{bmatrix}$$

6. Find the arc length of $y=\frac{2}{3}x^{\frac{3}{2}}$ from x=3 to x=8. [10%]

7. "Fermat's principle" states that the path taken between two points by a ray of light is the least-time path. Derive Snell's law using "Fermat's principle". [10%]

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8. Using separation of variables to solve the following partial differential equations [10%].

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2},\ u(0,t) = u(L,t) = 0,$$
$$u(x,0) = f(x),\ u_t(x,0) = 0,\ (0 \le x \le L).$$

9. Experiments show that at each instant a radioactive substance decays at a rate proportional to the amount present. Show that $\lambda T_{1/2} = \ln 2$, where λ is decay constant and $T_{1/2}$ is "half-life", period of time during which the radioactive substance decays to half. [10%]

10. Use the divergence theorem to evaluate the surface integral of $\vec{F} = [x, y, z]$, S the sphere of $x^2 + y^2 + z^2 = 9$. [10%]