

國立中央大學 107 學年度碩士班考試入學試題

所別： 地球科學學系地球物理 碩士班 不分組(一般生)

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地球科學學系地球物理 碩士班 不分組(在職生)

科目： 微積分

本科考試禁用計算器

*請在答案卷(卡)內作答

作答時須列出完整計算過程

1. (a) $\lim_{x \rightarrow 0^+} x^x = ?$ (5%)

(b) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan x - 5}{\sec x + 4} = ?$ (5%)

2. (a) $\tan \theta = \frac{y}{x}$, $\frac{\partial \theta}{\partial x} = ?$ (5%)

(b) $y = x^x$, $\frac{dy}{dx} = ?$ (5%)

3. (a) $\int_{-\pi}^{\pi} (\cos mx)(\cos nx) dx = ?$ (m, n : integers) (5%)

(b) $\int_{-\infty}^{\infty} e^{-x^2} dx = ?$ (5%)

4. (10%) Find the even periodic expansions of the function (half-range expansion)

$$f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2} < x < L. \end{cases}$$

5. (10%) Use Laplace transform to solve $\frac{\partial^2 w(x,t)}{\partial t^2} = c^2 \frac{\partial^2 w(x,t)}{\partial x^2}$, with two

boundary conditions (1) $w(0,t) = f(t) = \begin{cases} \sin t, & \text{if } 0 \leq t \leq 2\pi \\ 0 & \text{otherwise} \end{cases}$,

(2) $\lim_{x \rightarrow \infty} w(x,t) = 0$ ($t \geq 0$), and two initial conditions (1) $w(x,0) = 0$,

(2) $\frac{\partial w}{\partial t} \Big|_{t=0} = 0$.

注意：背面有試題



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6. (10%) Solve the wave equation $\frac{\partial^2 u(r,t)}{\partial t^2} = c^2 \nabla^2 u$ for the vibration of a circular membrane of radius R that satisfies the boundary condition $u(R,t) = 0$, for all $t \geq 0$, and two initial conditions $u(r,0) = f(r)$, $\frac{\partial u}{\partial t} \Big|_{t=0} = 0$.
7. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ (10%)
8. Solve the nonhomogeneous ODE $2y'' + 4y' + 6.5y = \cos 1.5t$ (10%)
9. "Fermat's principle" states that the path taken between two points by a ray of light is the least-time path. Derive Snell's law using "Fermat's principle". (10%)
10. Experiments show that at each instant a radioactive substance decays at a rate proportional to the amount present. Show that $\lambda T_{1/2} = \ln 2$, where λ is decay constant and $T_{1/2}$ is "half-life", period of time during which the radioactive substance decays to half. (10%)

注意:背面有試題

參考
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