作答時須列出完整計算過程

1. (a) \( \frac{d^2}{d\theta^2} \cos^{-1}\theta = ? \) (5%)

   \( \lim_{x \to 0} \frac{\sin 2x - 2x}{x^3} = ? \) (5%)

2. (a) \( \int x^3 \cos 3x dx \) (5%)

   (b) \( y = x^x, \quad \frac{dy}{dx} = ? \) (5%)

3. (10%) Solve the initial value problem

   \[ y'' + y' - 2y = 0, \quad y(0) = 4, \quad y'(0) = -5. \]

4. (10%) Find the eigenvalues and eigenvectors of the matrix

   \[ A = \begin{bmatrix} -4.0 & 4.0 \\ -1.6 & 1.2 \end{bmatrix} \]

5. (10%) Legendre functions are defined by

   \[ P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} [(x^2 - 1)^n]. \]

   Associated Legendre functions are defined by

   \[ P_k^m(x) = (1 - x^2)^{\frac{k}{2}} \frac{d^k P_m(x)}{dx^k}. \]

   Find \( P_0(x), P_1(x), P_2(x), P_1^1(x), P_2^2(x). \)
6. (10%) Find the Laplace transform of \( \cos \omega t, \sin \omega t, \) and \( e^{at} \cos \omega t \).

Find the Inverse Laplace transform of \( \frac{s+11}{s^2-2s-3} \).

7. (10%) Find the inverse of the matrix

\[
A = \begin{bmatrix}
-1 & 1 & 2 \\
3 & -1 & 1 \\
-1 & 3 & 4
\end{bmatrix}
\]

8. (10%) Find the integral of \( \int_{-\infty}^{\infty} e^{-x^2} \, dx \)

9. (10%) Find the Fourier coefficients of the periodic function \( f(x) \). The formula is

\[
f(x) = \begin{cases} 
-k & \text{if } -\pi < x < 0 \\
k & \text{if } 0 < x < \pi 
\end{cases}, \quad f(x + 2\pi) = f(x).
\]

10. (10%) Use the method of separating variables to solve the one-dimensional wave equation \( \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \) for the vibrations of an elastic string of length \( L \). The boundary conditions are \( u(0, t) = 0, u(L, t) = 0 \) for all \( t \). The initial conditions are \( u(x, 0) = f(x), u_t(x, t)|_{t=0} = g(x) \).