所別：地球物理研究所碩士班 科目：微積分

1.求下列極限: (8%)

(a) $\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x}$ (b) $\lim_{x\to\infty}\frac{(\log x)^n}{x}$

2.求下列積分: (16%)

(a) $\int x^2 e^x dx$ (b) $\int\frac{dx}{x^4+1}$ (c) $\int(\sin^2\phi+\cos\phi)^2 d\phi$

(d) $\int\frac{\cos mx}{(x-a)^2+b^2}dx$ $(m>0)$

3.求下列微分: (12%)

(a) $y=\tan(x)\sin(\cos(\sqrt{x^2+1}))$ (b) $y=x^{x^2}$ (c) $y=2\sin^{-1}(\frac{x}{2})-\frac{1}{2}x\sqrt{4-x^2}$

4.Find all the relative maxima and minima of the function,

$y=\frac{x-1}{x^2+3}$. (Hint: Use the second derivative test) (8%)

5.Solve the equation $y'=\frac{1+x}{xy}$, where y(1) = -2. (6%)

6.(1) Define the following terms: (8%)

(a) Real symmetric matrix (b) Hermitian matrix

(c) Orthogonal matrix (d) Unitary matrix

For clarity, please define and explain all the symbols being used in your answers.

(2)Show and determine the nature (using the terms a,b,c,d above) of the following matrices (12%)

(i) $\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}$ (ii) $\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}}\\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}$ (iii) $\begin{bmatrix}2 & 1-i\\ 1+i & 5\end{bmatrix}$

注意：背面有試題

(iv) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (v) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ (vi) $\begin{bmatrix} 1 & 3-i \\ 3+i & 2 \end{bmatrix}$

7.設一流場的流動速度場爲 $\vec{v} = 2x\vec{i} - 4y\vec{j} + 2z\vec{k}$

(1)求 $\nabla \bullet \vec{V}$ 及 $\nabla \times \vec{V}$

(2)求流場是否可壓縮? 流場是否可旋轉?

(3)若 $\vec{V} = \nabla\phi$ 求 $\phi(x, y, z) = ?$ (15%)

8.Consider a dynamic system $[M]\{\ddot{x}\} + [K]\{x\} = \{0\}$ (15%)

For which $[M] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $[K] = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$ and $\{x\} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$

(1) Derive the eigenvalue equation.

(2) Determine the eigenvalues and eigenvectors.