

所別：地球物理研究所碩士班 一般生 科目：微積分  
學位在職生

1. If a particle moves along a curve  $x=x(t)$ ,  $y=y(t)$  with constant speed  $v(t)=c$ , show that the magnitude of the acceleration is

$$|a(t)| = \frac{|x'y'' - x''y'|}{c} \quad (10\%)$$

2. 求下列積分: (20%)

(a)  $\int \frac{\sqrt{1-2u^{1/3}}}{u^{1/3}} du$ , (b)  $\int \frac{t dt}{\sqrt{2x^2+3}}$ , (c)  $\int (\sin^2 \phi + \cos \phi)^2 d\phi$

(d)  $\int_0^{\sqrt{3}} \frac{\cos^{-1} x}{1+x^2} dx$ .

3. 求下列微分: (20%)

(a)  $\cos^{-1} \sqrt{\frac{2a-y}{2a}}$ , (b)  $\tan^{-1} x + \tan^{-1}(\frac{1}{x})$ , (c)  $y = 4 \sin^{-1}(\frac{x}{2}) - \frac{1}{2} x \sqrt{4-x^2}$ ,

(d)  $\sec^{-1} \sqrt{x}$ .

4. For the two vectors

$\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$ ,  $\vec{B} = -2\hat{i} + 3\hat{j} + \hat{k}$  find (a)  $\vec{A} - \vec{B}$  and  $|\vec{A} - \vec{B}|$ , (b) component of  $\vec{B}$  along  $\vec{A}$ , (c) angle between  $\vec{A}$  and  $\vec{B}$ , (d)  $\vec{A} \times \vec{B}$ ,

(e)  $(\vec{A} - \vec{B}) \times (\vec{A} + \vec{B})$  (20%)

5. 設一流場的流動速度場為  $\vec{v} = 2x\hat{i} - 4y\hat{j} + 2z\hat{k}$ .

(1) 求  $\nabla \cdot \vec{v}$  及  $\nabla \times \vec{v}$

注意：背面有試題

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(2) 求流場是否可壓縮？流場是否可旋轉？

(3) 若  $\vec{v} = \nabla\phi$  求  $\phi(x, y, z) = ?$  (15%)

6. Taylor's theorem is relates to the expansion of an arbitrary function in a power series. Show that the Taylor series expansion of (1)  $e^x$ , (2)  $\sin x$  and  $\cos x$ , (3) Verify the result by expanding the integral in the relation  $\sin^{-1} x = \int_0^x \frac{dt}{\sqrt{1-t^2}}$ . (15%)