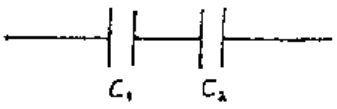
國立中央大學八十五學年度碩士班研究生入學試題卷

所別：太空科學研究所　不分組　科目：普通物理　共 1 頁 第 1 頁

(每題 20 分)

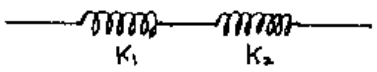
1. (A) Show that for two capacitors (電容器) with C_1 and C_2 in series (串聯), the equivalent capacitance C_{eq} is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

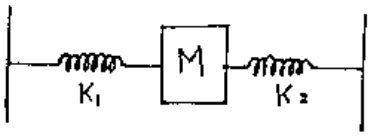


(B) Show that for two springs (彈簧) with force constants k_1 and k_2 in series, the equivalent force constant k_{eq} is

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$



(C) A block with mass M slides as shown on a frictionless track. It is constrained by two springs as shown. What is the equation of motion of the block and what is the frequency of the simple harmonic motion.

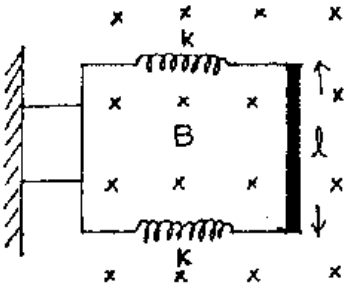


2. (A) Explain the 1st law and the 2nd law of thermodynamics.

(B) A heat engine operates between 200° K and 100° K. In each cycle it takes 100 J from the hot reservoir, loses 25 J to the cold reservoir and does 75 J of work. This heat engine violates (a) both the 1st law and the 2nd law of thermodynamics, (b) the 1st law but not the 2nd law of thermodynamics, (c) the 2nd law but not the 1st law of thermodynamics, (d) neither the 1st nor the 2nd law of thermodynamics, (e) cannot answer without knowing the mechanical equivalent of heat.

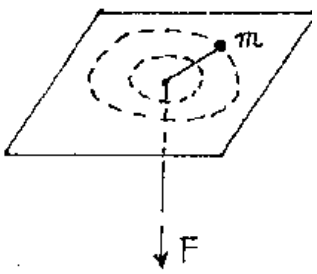
Which one of (a)~(e) is correct? Explain your reason clearly.

3. A conducting loop is formed with two springs (k = 2 N/m) and a rod of length l = 30cm and mass m = 20g, as shown in the figure. A uniform magnetic field of 0.4 T is directed perpendicular to the plane of the loop. At t = 0, the rod is released with the springs extended by A = 10cm. (A) Write an expression for the induced emf, $\xi(t)$. (B) What is the maximum value of the emf, and when does it occur for the first time?



4. The figure shows a particle of mass m moving in a circle with the centripetal force provided by a rope that passes through a hole in the table. The initial angular momentum is L_0. The force is changed in such a way that the radius of the motion decreases from r_1 to r_2. (A) How does the force vary as a function of r?

(B) Calculate the work done by the force in changing the radius. (C) What is the change in kinetic energy of the particle? (D) Does the work energy theorem apply?



5. A circular ring of radius R has a linear charge density λ (total charge Q).

(A) Show that the field strength along the axis at a distance x from the center is

$$E(x) = \frac{kQx}{\left(x^2 + R^2\right)^{3/2}}$$

(B) Find a simplified expression for $E(x)$ when $x << R$.

(C) Show that a charge $-q$ with mass m would undergo simple harmonic motion for small displacement from the center along the axis.

(D) Show that the angular frequency of the oscillation is

$$\omega = \sqrt{\frac{kqQ}{mR^3}}$$