1.　It is known:

e (electron charge) = 1.6×10^{-19} C,

m (electron mass) = 9.1×10^{-31} kg, *and*

ε_0 (vacuum permittivity) = 8.854×10^{-12} F/m.

And given conditions:

N (electron density) = 1.24×10^{12} electrons m^{-3}

B_0 (magnetic field) = 0.5×10^{-4} tesla,

Calculate the associated gyrofrequency and the plasma frequency. (15%)

2.　The refractive index of a radio wave of frequency *f* is given by

$$\mu^2 = 1 - \frac{k}{f^n \cdot \cos\theta}$$

where θ is the angle between the direction of phase propagation and a reference direction and *k* is a constant. Find the value of *n* for which the group speed v_g and the phase speed v_p are related by $v_p \times v_g = C^2$, where C is the speed of light. (20%)

3.　Assume that, in the ionosphere, the refractive index (μ) as a function of the height (z) and the radio frequency (*f*) is given by

$$\mu = 1 - \frac{1}{2}\left(\frac{z \cdot f_{NB}}{h \cdot f}\right)^2,$$

where *h* is the height at the point B, and f_{NB} is the plasma frequency at B. Find (1) the phase path (P), (2) the real path, and (3) the group path (P') between A and B as shown as the following figure, when $f = 3 f_{NB}$. (25%)
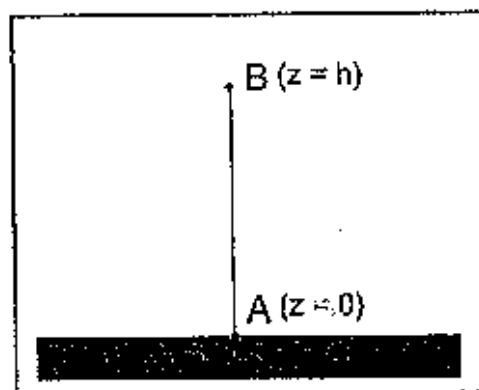
4. Assume that the ionosphere is composed by thin layers and has negligible magnetic field effects, collision damping, and the curvature on Earth's surface. Find the reflection height at various radio incidence angle ($\theta =0°$ and $30°$) and a radio frequency f of $0.5f_c$ ($f = 0.5f_c$) for the following mathematical model of the ionospheric electron density.

$$f_N^2 = f_C^2 \left[1 - \left(\frac{z - h_m}{y_m} \right)^2 \right]$$

where f_c is the critical frequency, h_m is the height at the maximum electron density, and y_m is the half thickness of this parabolic layer. Note that $y_m < h_m$. (20%)

5. (1) Plot an example of vertical ionogram showing E, F, and Es layer echoes and including both of ordinary and extraordinary traces, and
(2) present its corresponding scaled frequency and height parameters for the O-mode trace, e.g. foE, foF2, hoE, hoF2, etc. (20%)