

系所別:

太空科學研究所

科目:

應用數學

1. Find the envelope of the family of curves given by $y = cx + c^2$. (8%)
2. Use the method of undetermined coefficients to find the complete solution of the differential equation: $\frac{d^2 y}{dx^2} - \frac{dy}{dx} = 2x + 1 - 4 \cos x + 2e^x$. (12%)
3. (a) Write down the definition of Gamma function $\Gamma(x)$. (7%)
(b) Evaluate the value of $\Gamma(\frac{1}{2})$. (13%)
4. If $x = r \cos \theta$ and $y = r \sin \theta$, express each of the following as function of r and θ :
- (a) $(\frac{\partial \theta}{\partial r})_y$ (5%).
(b) $(\frac{\partial r}{\partial \theta})_z$ (5%).
5. (a) In circular cylindrical coordinate (R, θ, z) , let $\vec{F} = \frac{1}{R} \hat{e}_\theta$, show that $\vec{\nabla} \times \vec{F} = 0$. (5%)
(b) In spherical coordinate (r, θ, ϕ) , let $\vec{F} = r^n \vec{r}$, show that $\vec{\nabla} \cdot \vec{F} = (n+3)r^n$ (5%)
6. (a) Evaluate the value of line integral: $\oint_c (\frac{1}{R} \hat{e}_\theta) \cdot d\vec{l}$,
where $c: \frac{x^2}{1} + \frac{y^2}{4} + \frac{z^2}{9} = 1, z = 0$. (10%).
(b) Evaluate the value of surface integral: $\iint_\Sigma (r^{-3} \vec{r}) \cdot d\vec{s}$,
where $\Sigma: \frac{x^2}{1} + \frac{y^2}{4} + \frac{z^2}{9} = 1$. (10%).
7. Use residue calculus to evaluate the following integrals:
- (a) $\int_0^{2\pi} \frac{d\theta}{A + B \cos \theta}$, ($A^2 > B^2, A < 0$). (10%).
(b) $\int_{-\infty}^{\infty} \frac{\sin x}{x} dx$, (10%).

參考