國立中央大學99學年度碩士班考試入學試題卷

所別：太空科學研究所碩士班 不分組(一般生)　科目：應用數學　共 1 頁 第 1 頁

本科考試禁用計算器

＊請在試卷答案卷（卡）內作答

(1, 20%) Find the general solutions of the following ordinary differential equations:

(a) $\left(x^2-1\right)\frac{dy}{dx}=xy$ (10%)　　(b) $\frac{dy}{dx}+2y=\cos x$ (10%).

(2, 20%) Derive the following formulas involving del operators ($\nabla=\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}$):

(a) $\nabla\times(U\mathbf{A})=(\nabla U)\times\mathbf{A}+U(\nabla\times\mathbf{A})$ (10%)

(b) $\nabla\times(\nabla\times\mathbf{A})=\nabla(\nabla\cdot\mathbf{A})-\nabla^2\mathbf{A}$ (10%),

where U is the scalar function and $\mathbf{A}$ the vector function of x, y, and z, and has partial derivatives.

(3, 20%) Derive the scale factors, h_r, h_θ, and h_ϕ of a spherical coordinate (r, θ, ϕ). It is noted that the differential of arc length ds is determined from $ds^2=h_r^2dr^2+h_\theta^2d\theta^2+h_\phi^2d\phi^2$.

(4, 20%) Find the inverse of $\begin{bmatrix}1 & 2 & 1\\ 2 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}$ with Gauss-Jordan elimination and check the result.

(5, 20%) Find the general solutions of the following partial differential equations:

(a) $\frac{\partial^2 z}{\partial x^2}-3\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=0$ (10%)　　(b) $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=0$ (10%).