## 國立中央大學99學年度碩士班考試入學試題卷

所別:太空科學研究所碩士班 不分組(一般生) 科目:近代物理 共 / 頁 第 / 頁 本科考試禁用計算器 \*請在試卷答案卷(卡)內作答

- 1. A satellite transmits signals of wavelength 24cm in its frame.
- (a) As it moves along the line connecting it and Earth, observers on Earth receive wavelength 24.4 cm. What is the satellite velocity? (5%)
- (b) If the satellite moves in the opposite direction at the same speed as in (a). What the wavelength would be seen? (5%)
- (c) If the satellite moves in circle around Earth at the same speed. What the wavelength would be seen? (10%)
- 2. For two observers, the observer S' moves at velocity  $\nu$  relative the observer S in the positive x-direction. A radio signal moves at an angle  $\theta$  with the x-direction as seen from the observer S. Using the relativistic velocity transformation, find the components of the velocity of the signal when viewed from the observer S'. (10%)
- 3. A proton is usually regarded as being of radius  $10^{-6}$  nm.
- (a) If the kinetic energy of the proton is equal to its rest energy, would the proton behave as a wave or as a particle? (5%)
- (b) If the kinetic energy of the proton has 0.5 of its rest energy, would the proton behave as a wave or as a particle? (5%)
- 4. Why do we need to consider the quantized characteristics of systems in which particles are bound in a small region? (10%)
- 5. By regarding the theory of light passing through a transparent thin film of width d as a quantum particle of energy E encounters a potential barrier of height U where E>U.
- (a) Is it possible to make one that reflects no light? Why? (5%)
- (b) Is it possible to make one that transmits no light? Why? (5%)
- 6. The electron in a hydrogen atom is in a state of wavefunction in spherical coordinates  $(r, \theta, \phi)$  as

$$\Psi(r,\theta,\phi) = \frac{A}{8\sqrt{\pi}(a_0)^{3/2}} \frac{r}{a_0} e^{-r/(2a_0)} \left[ \sqrt{2}\cos\theta - (\sin\theta)(e^{i\phi}) + (\sin\theta)(e^{-i\phi}) \right],$$
where  $a_0$  is the Bohr radius and  $A$  is a possible of  $A$  in  $A$ .

where  $a_0$  is the Bohr radius and A is a normalized constant

- (a) Find the expectation value of the z-component of angular momentum. (5%)
- (b) Find the expectation value of the total angular momentum. (5%)
- (c) Find the expectation value of the energy of the state. (5%)
- (d) Find the uncertainty of the z-component of angular momentum. (5%)
- 7. The spin-orbit interaction splits the hydrogen 3d energy level into many states.
  - (a) Rank these states in order of increasing energy.
  - (b) If a weak external magnetic field were introduced without breaking the spin-orbit interaction. How many energy levels are split into? (5%)
- 8. A beam of helium in their ground state is sent through a Stern-Gerlach apparatus.
  - (a) What would you expect to see? (5%) (b) Why? (5%)

