## 國立中央大學99學年度碩士班考試入學試題卷

## 所別:太空科學研究所碩士班 不分組(一般生) 本科考試禁用計算器

- 1. Answer the following 12 questions briefly:
  - (a) What is the physical meaning of Gauss's law for electric fields. (5 points)
  - (b) How do we define a vector field conservative? (5 points)
  - (c) If there is no surface charge at a boundary, what is the relation between the normal components of the electric flux density on both sides of the boundary? (5 points)
  - (d) What does it mean when we say that a medium is isotropic? (5 points)
  - (e) Write down the charge continuity equation in differential form and explain its physical meaning. (5 points)
  - (f) why is the electric field intensity created by a charged particle in free space is larger than that in a dielectric medium. (5 points)
  - (g) Are the normal components of the magnetic flux density on both sides of a boundary always the same? (5 points)
  - (h) How do we calculate the energy in a magnetic field? (5 points)
  - (i) How does an electric generator work? (5 points)
  - (j) What term in Maxwell's equations enabled James C. Maxwell discover that electromagnetic fields should propagate as waves? (5 points)
  - (k) What kind of medium can make propagating electromagnetic waves losing energy? (5
  - (l) What is the electromagnetic mechanism behind an electric motor? (5 points)
- 2. Use the generalized Ohm's law with finite conductivity  $(\vec{E} + \vec{u} \times \vec{B} = \eta \vec{J})$ , Faraday's law  $(\nabla \times \vec{\mathbf{E}} = -\partial \vec{\mathbf{B}}/\partial t)$ , and Ampere's law  $(\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}})$  to derive

$$\frac{\partial \vec{\mathbf{B}}}{\partial t} = \nabla \times (\vec{\mathbf{u}} \times \vec{\mathbf{B}}) + (\eta/\mu_0) \nabla^2 \vec{\mathbf{B}}$$

where  $\vec{\mathbf{B}}$  is the magnetic flux density,  $\vec{\mathbf{E}}$  is the electric field intensity, and  $\vec{\mathbf{u}}$  is the velocity of plasma;  $\eta$  is the resistivity and  $\mu_0$  is the permeability in free space. The first term on the right-hand side describes the convection of the field with the plasma, while the second term describes the diffusion of the field across the plasma. (20 points)

Useful vector identity:  $\nabla \times \nabla \times \vec{A} = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$ 

- 3. The motion of a charged particle is governed by the Lorentz Force equation  $(\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}})$ , where  $ec{\mathbf{B}}$  is the magnetic flux density,  $ec{\mathbf{E}}$  is the electric field intensity, and q is the charge and  $ec{\mathbf{v}}$  is the velocity of the particle. With the equation, discuss the possible trajectories and the velocity change of a negative charge moving with a non-zero initial velocity  $\vec{\mathbf{v_0}}$  under the following conditions of the electric and magnetic fields:
  - (a) No electric and magnetic fields, (5 points)
  - (b) Uniform magnetic fields only, (7 points)
  - (c) Uniform electric fields only. (8 points)

