

國立中央大學100學年度碩士班考試入學試題卷

所別：水文與海洋科學研究所碩士班不分組(一般生)

科目：應用數學

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本科考試禁用計算器

*請在試卷答案卷(卡)內作答

1. (a) [10%] What is the Taylor series of a real function $f(x)$? What are the conditions that the Taylor series of a real function $f(x)$ exists?

(b) [10%] Expand $f(x) = \sqrt[4]{x}$ as a Taylor series about $x_0 = 16$.

(c) [10%] Use the Taylor series in (a) to three terms to find approximation of $\sqrt[4]{17}$ to four decimal places.

2. [20%] Evaluate the double integral $I = \iint_D \sin y \, dx \, dy$, where D is the triangular region bounded by the lines $2y = x$, $y = 2x$ and $x = \pi$.

3. [20%] If $x = e^u \cos v$ and $y = e^u \sin v$, show that

$$\frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2} = (x^2 + y^2) \left(\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} \right),$$

where $F(x, y) = G(u, v)$.

4. (a) [20%] A solution of the differential equation:

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = 4e^{-x}$$

takes the value 1 when $x = 0$ and the value e^{-1} when $x = 1$. What is its value when $x = 2$

(b) [10%] Is the differential equation in (a) linear or nonlinear, homogeneous or nonhomogeneous?

參考用