所別：水文與海洋科學研究所碩士班 不分組（一般生）　　科目：應用數學　共 1 頁　第 1 頁
　　　水文與海洋科學研究所碩士班 不分組（在職生）

本科考試禁用計算器　　　　　　　　　　　　　　　　　　　＊請在試卷答案卷（卡）內作答

1. Find the following derivatives,

   (a)[10%] Given $y = \dfrac{2-(1/x)}{x+4}$, find $\dfrac{dy}{dx}$

   (b) [10%] Given $x^2 + y^2 = 20$, find $\dfrac{d^2 y}{dx^2}$

   (c) [10%] Given $y = x^{3x}$, x >0, find $\dfrac{dy}{dx}$

2. Evaluate the following integrations

   (a) [10%] $\displaystyle\int \dfrac{1}{1+e^x} dx$

   (b) [10%] $\displaystyle\int_{1}^{3/2} x\sqrt{3-2x}\,dx$

3. Given two vectors, $U = \hat{i} - 2\hat{j} + 3\hat{k}$, $V = 3\hat{i} - 2\hat{j} + \hat{k}$,

   find (a) [5%] dot product of $U \bullet V$, (b) [5%] cross product of $U \times V$

4. Let $F = x^2 yz\,\hat{i} + xy^2 z\,\hat{j} + xyz^2\,\hat{k}$, find (a) [5%] $div\,F$, (b) [5%] $curl\,F$

5. Solve the initial value problems,

   (a) [5%] $2yy' + x = 0$, $y(0) = 1$, (b) [5%] $y' + y = 4$, $y(1) = 2$

6. [10%] Solve the differential equations, $x^2 - y^2 = -2xyy'$

7. [10%] Find the eigenvalues and eigenvectors of the following A matrix

   $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$