

國立中央大學 105 學年度碩士班考試入學試題

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科目： 微積分

本科考試禁用計算器

*請在答案卷(卡)內作答

1. Find the following derivatives,

(a) [10%] Given $y = x^3 \cos(\ln x)$, find $\frac{dy}{dx}$

(b) [10%] Given $e^{(x+y)} = x^3$, find $\frac{dy}{dx}$ in terms of x and y

(c) [10%] Given $y \sin(y) = x$, find $\frac{d^2y}{dx^2}$ in terms of x and y

2. [15%] Find the Maclaurin series of $\ln\left(\frac{1}{1-x}\right)$ to four terms. (Maclaurin series is a Taylor series expansion of a function about 0)

3. Evaluate the following integrations:

(a) [10%] $\int \frac{x-3}{x^2-5x+4} dx$

(b) [10%] $\int_0^\pi x^2 \cos(x) dx$

4. [15%] Show that

$$\int_0^{2\pi} \sin(mt) \sin(nt) dt = \pi \delta_{mn}$$

where

$$\delta_{mn} = \begin{cases} 0 & \text{for } m \neq n \\ 1 & \text{for } m = n \end{cases}$$

for any positive integers m and n

5. Given two vectors, $\mathbf{U} = 3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$, and $\mathbf{V} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ find

(a) [5%] dot product of $\mathbf{U} \cdot \mathbf{V}$

(b) [5%] cross product of $\mathbf{U} \times \mathbf{V}$

(c) [10%] the area of triangle formed by \mathbf{U} and \mathbf{V}