

# 國立中央大學 112 學年度碩士班考試入學試題

所別： 企業管理學系碩士班

共 5 頁 第 1 頁

科目： 統計學      計算題應詳列計算過程，無計算過程者不予計分

1. Assume  $X$  follows a normal distribution with mean  $\mu$  and variance 25. To test  $H_0: \mu = 50$  against  $H_1: \mu > 50$ , let the rejection region be defined by  $C = \{\bar{X} > 51.5\}$ , where  $\bar{X}$  is the sample mean of a random sample of size  $n = 25$  from this distribution.
  - (a) What is the significant level of this test? (4 pts)
  - (b) What is the  $p$ -value corresponding to  $\bar{x} = 51.28$ ? (4 pts)
  - (c) Let  $K(\mu)$  be the power function for this test. For example,  $K(53)$  represents the testing power if  $\mu = 53$ . Please find the values of  $K(50)$ ,  $K(51.5)$ , and  $K(53)$ ? (6 pts)
  - (d) What is the probability of making a type II error if the alternative hypothesis is set to be  $H_1: \mu = 53$ ? (4 pts)
  - (e) Following (d), (to test  $H_0: \mu = 50$  against  $H_1: \mu = 53$ ), if we want to reduce the probability of making a type II error by a half while achieving the same significant level in (a), please determine the (minimal) required sample size,  $n$ , and the appropriate rejection region. (12 pts)
2. Suppose a civil engineer wishes to compare the strengths of two different types of beams, one (A) made of steel and the other (B) made of a different and more expensive alloy. A certain deflection (in units of 0.001 inch) was measured for each beam when submitted to a given force; thus a small deflection would indicate a beam of great strength. The order observations for the two samples of size  $n_1 = 8$ , and  $n_2 = 6$  are the following:

Types	Deflection (in 0.001 inch)							
A	79	81	83	84	86	86	86	87
B	74	75	76	77	78	82		

- (a) Please conduct a 2-samples  $t$ -test to identify whether there is a difference between the two means of deflections of different types of beams? Use  $\alpha = 0.05$ . (10 pts)
  - (b) Please try to use ANOVA ( $F$ -test) to identify whether there is a difference between the two means of deflections of different types of beams? Use  $\alpha = 0.05$ . (8 pts)
  - (c) Show the two tests in (a) and (b) are equivalent. (2 pts)
3. Following problem 2, suppose there are two more different types of beams, (C) and (D), made of different alloys. The resulted sample means and sample variances of deflections from the two corresponding samples of Type C with size  $n_3 = 6$ , and of Type D with size  $n_4 = 6$ , are as following:

Types	Sample size	Sample mean	Sample Variance
C	6	79	7.6
D	6	73	8.0

- (a) Please conduct an ANOVA to identify whether there is a difference between the four means of deflections of different types of beams? Use  $\alpha = 0.05$ , and summarize the test result in an ANOVA table. (10 pts)

注意：背面有試題

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科目： 統計學

- (b) Following the test result in (a), we need to make comparisons between the four means, noted as pairwise- $t$  tests. Please conduct a pairwise- $t$  test between the two means of deflections of type A and type B with  $\alpha = 0.05$ . Show the difference between problems 2(a) and 3(b). (10 pts)
4. Following 10 records are used to train a Naïve Bayesian Classification model to predict tennis-playing or not, based on weather information.

Day	Outlook	Temperature	Humidity	Wind	Play Tennis
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
3	Overcast	Hot	High	Weak	Yes
4	Rainy	Mild	High	Weak	Yes
5	Rainy	Cool	Normal	Weak	Yes
6	Rainy	Cool	Normal	Strong	No
7	Overcast	Cool	Normal	Strong	Yes
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
10	Rainy	Mild	Normal	Weak	Yes

- (a) Let's define the event  $A$  as "play\_tennis = yes", while  $A^C$  as "play\_tennis = no". Please determine the (prior) probabilities of  $\Pr(A)$  and  $\Pr(A^C)$  by counting the frequencies based on the 10 records. (4 pts)
- (b) Let's define the event  $B_1$  as "outlook = sunny",  $B_2$  as "outlook = overcast", and  $B_3$  as "outlook = rainy". Please determine the conditional probabilities of  $\Pr(B_i|A)$  and  $\Pr(B_i|A^C)$ ,  $i = 2, 3$ , by counting the frequencies based on the 10 records. (for example,  $\Pr(B_1|A) = 1/6$  and  $\Pr(B_1|A^C) = 3/4$ ) (8 pts)

Similarly, we can define events

$C_1$  as "temperature = hot",  $C_2$  as "temperature = mild", and  $C_3$  as "temperature = cool";

$D_1$  as "humidity = high", and  $D_2$  as "humidity = normal";

$E_1$  as "wind = weak", and  $E_2$  as "wind = strong".

Assume events  $\{B_i\}$ ,  $\{C_j\}$ ,  $\{D_k\}$ , and  $\{E_l\}$  are mutually independent. Hence the  $\Pr(B_i \cap C_j \cap D_k \cap E_l)$  can be determined as  $\Pr(B_i) \cdot \Pr(C_j) \cdot \Pr(D_k) \cdot \Pr(E_l)$ . For example,

$$\begin{aligned} & \Pr(\text{"outlook = sunny", "temperature = cool", "humidity = normal", "wind = weak"}) \\ &= \Pr(\text{"outlook = sunny"}) \cdot \Pr(\text{"temperature = cool"}) \cdot \Pr(\text{"humidity = normal"}) \cdot \Pr(\text{"wind = weak"}) \\ &= \frac{4}{10} \cdot \frac{4}{10} \cdot \frac{5}{10} \cdot \frac{7}{10} = 0.056 \end{aligned}$$

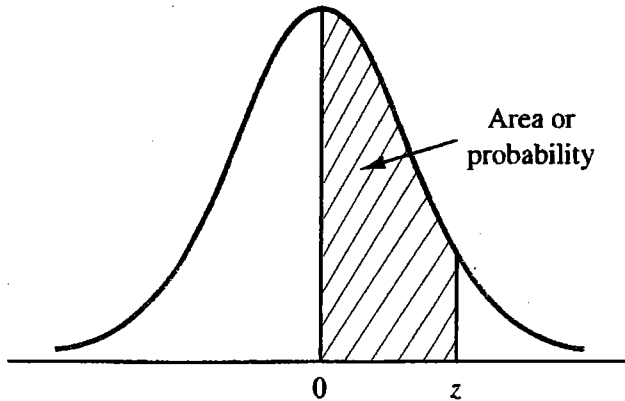
- (c) Let's use event  $X$  as "outlook = sunny, temperature = cool, humidity = normal, wind = weak". Please determine the conditional probabilities of  $\Pr(X|A)$  and  $\Pr(X|A^C)$  by counting the frequencies based on the 10 records with assuming independence. (4 pts)
- (d) Try to use Bayes' Theorem to calculate the (posterior) probabilities,  $\Pr(A|X)$  and  $\Pr(A^C|X)$ . (10 pts)
- (e) With the posterior probabilities in (d), would the unknown sample  $X$  be classified/predicted as "play\_tennis = yes" or "play\_tennis = no"? (4 pts)

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Entries in the table give the area under the curve between the mean and  $z$  standard deviations above the mean. For example, for  $z = 1.25$  the area under the curve between the mean and  $z$  is .3944.

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4986	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

注意：背面有試題

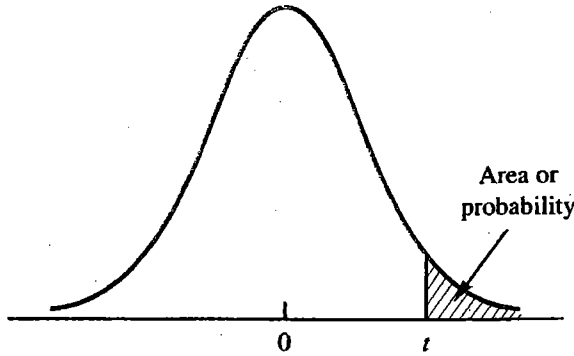
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共5頁 第4頁

科目：統計學

**t DISTRIBUTION**



Entries in the table give  $t$  values for an area or probability in the upper tail of the  $t$  distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail,  $t_{.05} = 1.812$ .

Degrees of Freedom	Area in Upper Tail				
	.10	.05	.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

注意:背面有試題

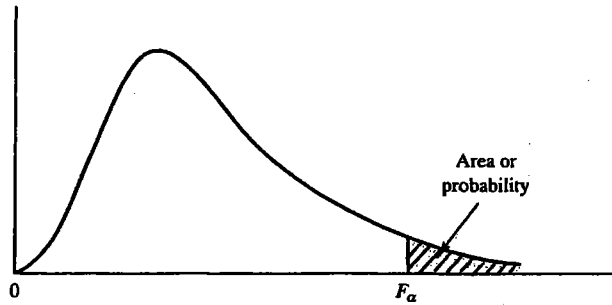
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科目： 統計學

**F TABLE - I**



Entries in the table give  $F_{\alpha}$  values, where  $\alpha$  is the area or probability in the upper tail of the  $F$  distribution. For example, with 12 numerator degrees of freedom, 15 denominator degrees of freedom, and a .05 area in the upper tail,  $F_{.05} = 2.48$ .

Table of  $F_{.05}$  Values

Denominator Degrees of Freedom	Numerator Degrees of Freedom																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00