1. Let A be an n by n matrix over real numbers. Prove or disprove that the eigenvalues of A equal the eigenvalues of A^T, where A^T is the transpose of A. (10%)

2. Let $\sum_{k=1}^{\infty} a_k$ be a convergent series with real numbers a_k. Prove or disprove that $\lim_{n \to \infty} |a_n| = 0$. (10%)

3. For $x > 0$, find the derivative of $f(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}$. (10%)

4. Let f be the function defined by
$$f(x) = \begin{cases} 0 & \text{if } x \text{ is an integer} \\ 1 & \text{if } x \text{ is a real number but not an integer.} \end{cases}$$

   (a) Is f continuous at $x = 1$? Explain your answer. (10%)

   (b) Find $\int_0^{100} f(x)\, dx$. Explain your answer. (15%)

5. Prove or disprove that the polynomial $P(x) = x^n + 100x + 1000$ of degree n cannot have more than three real roots if n is odd. (15%)

6. Evaluate $\int_0^3 |y^3 - 2|\, dy$. (10%)

7. Solve the differential equation $y'' + 2\sqrt{3}\, y' + 3y = 0$. (10%)

8. Find the extrema of the function g defined by real variables x and y, where $g(x, y) = x^3 - 12xy + 8y^3$. (10%)