所別：企業管理學系碩士班 一般類組(甲組) 科目：微積分

1. (21%) Find the following limit respectively.
   (a) $\lim_{x \to \infty} (5 + 6x^2)^{\frac{2}{\ln x}}$
   (b) $\lim_{n \to \infty} [\frac{1}{3n+1} + \frac{1}{3n+2} + \cdots + \frac{1}{4n}]$
   (c) $\lim_{n \to \infty} \frac{\int_0^\infty x^n e^{-x} dx}{n!}$

2. (10%) Let f satisfy

$$3f(x) + \int_0^x e^{t^2} f'(t) \, dt = x + 3 \ \forall x \in R.$$

   Find $f'(0)$ and an equation for the tangent line T to the graph of f at $x_0 = 0$.

3. (14%) In the following $(a) - (b)$, find the interval of convergence respectively.
   (a) $\sum_{k=1}^\infty \frac{(-1)^k}{k^3 4^k} (x + 3)^k$
   (b) $\sum_{k=0}^\infty \frac{2k^3}{e^k} x^k$

4. (15%) Let $f : R^2 \to R$ be defined by by

$$f(x, y) = \begin{cases} \sqrt{x^2 + y^2} \sin(\frac{y^2}{x}) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

   (a) Show that f is continuous at the point $(0, 0)$.
   (b) Show that f has directional derivatives in every direction at $(0, 0)$.
   (c) Is f differentiable at $(0, 0)$? Why ?

5. (15%) Find the following integral respectively.
   (a) $\int (\frac{e^x}{x} + e^x \ln x) \, dx$
   (b) $\int_1^\infty \frac{\ln x}{x^2} \, dx$
   (c) $\iint_A x^2 e^{y^4} dx dy$, where $A = \{(x, y) | \ 0 \leq x \leq 1, x \leq y \leq 1\}$.

6. (10%) Sketch the graph of $f(x) = x^2 e^x$, showing maximum points, minimum points, and points of inflection.

7. (a) (10%) Let $g(x, y) = x^4 - 2x^2 + y^2 - 2$. Find the local extreme values of g.
   (b) (5%) Determine the maximum value of $f(x, y, z) = 3x - 2y + z$ on the sphere $x^2 + y^2 + z^2 = 14$.