所別：企業管理學系碩士班 企業電子化組(庚組) 科目：離散數學

【1】Permutations and Combinations

(1.1) [5 Points] At high school, the gym teacher must make up four volleyball teams of nine boys each from the 36 freshman boys in her class. In how many ways can she select these four teams? Call the A, B, C and D.

(1.2) [5 Points] Suppose that Ellen draws five cards from a standard deck of 52 cards. In how many ways can her selection that contains at least one diamond?

【2】Set Theory

Let A, B, C are universal sets. Prove or disprove (with an example) each of the following:

(2.1) [5 Points] $A-C=B-C \Rightarrow A=B$

(2.2) [5 Points] $[(A\cap C=B\cap C)\text{ AND }(A-C=B-C)]\Rightarrow A=B$

(2.3) [5 Points] $[(A\cup C=B\cup C)\text{ AND }(A-C=B-C)]\Rightarrow A=B$

【3】Mathematical Induction

(3.1) [7 Points] Using the principle of mathematical induction to prove that for each $n\in Z^+$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

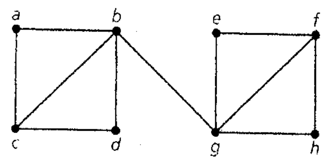
(3.2) [8 Points] Let $n\in Z^+$ where $n\geq 2$, and let $A_1, A_2, \ldots, A_n$ are universal sets for each $1\leq i\leq n$. Then $\overline{A_1\cap A_2\cap\cdots\cap A_n} = \overline{A_1}\cup\overline{A_2}\cup\cdots\cup\overline{A_n}$

【4】[10 Points] Recurrence Relations

Solve the relation $a_n-3a_{n-1}=n,\ n\geq 1,\ a_0=1$.

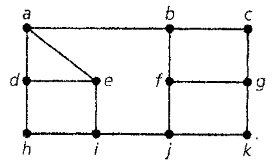
【5】Graph Theory with Applications

(5.1) [7 Points] Let G=(V, E) be the undirected graph. How many paths are there in G from *a* to *h* ? How many of these paths have length 5 ?



注意：背面有試題

(5.2) [8 Points] Let G=(V, E) be an undirected graph representing a section of a department store. The vertices indicate where cashiers are located; the edges denote unblocked aisles between cashiers. The department store wants to set up a security system where guards are placed at certain cashier locations so that each cashier either has a guard at his or her location or is only one aisle away from a cashier who has a guard. What is the smallest number of guards needed ?



【6】 [10 Points] Trees

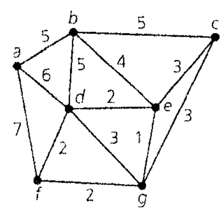
Please give an example to explain the following lemma:

Let *G=(V, E)* be a loop-free connected undirected graph with *T=(V, W)* a depth-first spanning tree of G. If *r* is the root of *T*, then *r* is an articulation point of G <u>**if and only if**</u> *r* has at least two children in *T*.

【7】 The Algorithms of Kruskal and Prime

(7.1) [5 Points] State the basic idea of Kruskal's algorithm.

(7.2) [10 Points] Apply Prim's algorithm to determine minimal spanning tree for the following graph.



【8】 [10 Points] The Max-Flow Min-Cut Theorem

Prove or give an example to explain the following statement:

For a transport network *N=(V, E)*, the maximal flow value that can be attained in *N* is equal to the minimal capacity over all cuts in the network.