國立中央大學98學年度碩士班考試入學試題卷

所別：企業管理學系碩士班 一般類組(甲組)　科目：微積分　共 1 頁 第 1 頁

＊請在試卷答案卷 (卡) 內作答

甲、填充題：共 8 題，每題 8 分，共 64 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. Evaluate $\int \frac{xe^{2x}}{(2x+1)^2}\,dx$. Answer : _____

2. What is the largest value that the directional derivative of $f(x,y,z)=xyz$ can have at the point $(1,1,1)$? Answer : _____

3. Evaluate $\int_0^1 \int_{\sqrt[3]{y}}^1 \frac{2\pi \sin \pi x^2}{x^2}\,dxdy$. Answer: _____

4. Assuming the equation $2xy+e^{x+y}-2=0$ define y as a differentiable function of x, find the value of dy/dx at point $P(0,\ln 2)$. Answer : _____

5. Find the area of the region that lies inside the circle $r=1$ and outside the cardioid $r=1-\cos\theta$. Answer : _____

6. Evaluate $\lim_{n\to\infty}\sum_{k=1}^{n} \ln \sqrt[n]{1+\frac{k}{n}}$. Answer : _____

7. A company estimates that the cost (in dollars) of producing x units of a product can be modeled by $C=800+0.04x+0.0002x^2$. Find the production level that minimizes the average cost per unit. Answer : _____

8. Find the length of the curve $y=\int_0^x \sqrt{\cos 2t}\,dt \quad 0\le x\le \pi/4$. Answer : _____

乙、計算、證明題：共 3 大題，每大題 12 分 (每小題 6 分)，共 36 分。須詳細寫出演算過程，否則不予計分。

1. Find the limits.

(a) $\lim_{k\to\infty} A_0\left(1+\frac{r}{k}\right)^{kt}$.　　(b) $\lim_{n\to\infty}\frac{2x}{x+7\sqrt{x}}$.

2. (a) How many solutions does the equation $\sin^2 t-3t=5$ have? Give reasons for your answer.

(b) If the graphs of f and g having inflection points at $x=a$, does the graph of $h=fg$ have an inflection point at a? Give reasons for your answer.

3. (a) Suppose that the function f and g are defined throughout an open interval containing the point x_0, that f is differentiable at x_0, that $f(x_0)=0$, and that g is continuous at x_0. Show that the product fg is differentiable at x_0.

(b) Show that $h(x)=\begin{cases} x^2\sin\frac{1}{x}, & \text{if } x\neq 0 \\ 0, & \text{if } x=0. \end{cases}$ is differentiable at $x=0$.