國立中央大學九十學年度碩士班研究生入學試題卷

所別: 資訊管理學系 丁組 科目: 離散數學 共 / 頁 第 / 頁

1. Prove that for all $n \in Z^+$, $n > 4 \Rightarrow 2^n > n^2$. (15%)

2. How many strings of length 20 on the letters M, I, S are there if the letter M occurs 10 times, the letter I occurs 5 times and the letter S occurs 5 times? (10%)

3. Prove that $\sqrt{5}$ is irrational. (15%)

4. Negate the following sentence. (10%)

   $$\forall x \exists y \forall z (P(x) \rightarrow Q(y, z))$$

5. Let $A = Z^+ \times Z^+$. Define a binary relation R on A as follows: for all (a, b) and (c, d) in A,

   $$(a, b) R (c, d) \Leftrightarrow a + d = c + b.$$

   Determine whether R is an equivalence relation. (10%)

6. An n-digit quaternary sequence consists of n quaternary digits $(0, 1, 2, 3)$. Let a_n be the number of n-digit sequences in which there is never a 3 anywhere to the right of a 0. Find and solve the recurrence relation for a_n. (10%)

7. Solve the following recurrence relation by the method of characteristic equation. (10%)

   $$a_{n+2} - 4a_{n+1} + 4a_n = 8 \times 2^n, n \geq 0; a_0 = 1, a_1 = 6.$$

8. Solve the following recurrence relation by the method of generating function. (10%)

   $$a_{n+2} - 4a_{n+1} + 3a_n = 2^n, n \geq 0; a_0 = 2, a_1 = 5.$$

9. Let $G = (V, E)$ be an undirected connected loop-free graph. Suppose further that G is planar and determines 53 regions. If, for some planar embedding of G, each region has at least 5 edges in its boundary, prove that $|V| \geq 82$. (10%)