1. Show whether each of the following arguments is valid. (10%)

    (a) If Ron can program in Java, then he can get the job.
    Ron does not get the job.
    Therefore, Ron can not program in Java.

    (b) If interest rates fall, then the stock market will rise.
    Interest rates are not falling.
    Therefore, the stock market will not rise.

2. Let $H_m = 1 + \dfrac{1}{2} + \dfrac{1}{3} + \cdots + \dfrac{1}{m}$. Prove that $1 + n/2 \le H_{2^n}$ for all $n \in N$. (10%)

3. Find the number of ways to assign 4 jobs to 3 workers so that each job gets a worker and each worker gets at least one job. (10%)

4. Suppose that we have 10 different pairs of shoes. From the 20 shoes, 4 are chosen at random. What is the probability of getting at least one pair? (10%)

5. Find the smallest positive integer a for which $2a$ is a perfect square and $3a$ is a perfect cube. (10%)

6. (10%)

    (a) Find a recurrence relation for the number of ways to climb a stair of n steps if the person climbing the stair can only take one or two steps going up at a time.

    (b) What are the initial conditions?

    (c) How many ways can this person climb a stair of 8 steps?

7. Find the smallest relation containing the relation {(1,2), (1,4), (3,3), (4,1)} that is reflexive, symmetric, and transitive. (10%)

8. Find the smallest equivalence relation on the set {a, b, c, d, e} containing {(a,b), (a,c), (d, e)}. (10%)

9. A tournament is a simple directed graph such that if u and v are distinct vertices in the graph, exactly one of (u, v) and (v, u) is an edge of the graph. How many different tournaments are there with n vertices? (10%)

10. How many different Boolean functions $F(x,y,z)$ are there so that $F(x,y,z) = F(\bar{x},\bar{y},\bar{z})$ for all values of the Boolean variables x, y, and z? (10%)