所別：資訊管理學系碩士班 丙組 科目：離散數學

一、複選題（40%）

答題說明：　(1) 每一題答案個數不等，可能為 0～6 個，全部答對得五分，否則得零分。

(2) 如果考生認為該題沒有答案（亦即答案個數為 0 個），則請回答「沒有答案」。

(3) 本試題不能也不需使用計算器，簡單的運算請以人工計算方式完成後答題。

1. For the machines in Table 1, which of the following statement(s) is/are true?
(a) There are no transient states (b) State s_4 is a sink state (c) There are three sub-machines (where I_1 = {0, 1}) (d) the only strongly connected sub-machines (where I_1 = {0, 1}) is { s_6}

Table 1

	ν		ω	
	0	1	0	1
s_0	s_1	s_2	0	1
s_1	s_0	s_2	1	1
s_2	s_2	s_3	1	1
s_3	s_6	s_4	0	0
s_4	s_5	s_5	1	0
s_5	s_3	s_4	1	0
s_6	s_6	s_6	0	0

2. Which of the following statement(s) is/are false?
(a) $\phi \in \{\phi,\{\phi\}\}$ (b) $\{\phi\} \in \{\phi\}$ (c) $\{\phi\} \in \{\{\phi\}\}$ (d) $\{\phi\} \subset \{\phi,\{\phi\}\}$
(e) $\{\{\phi\}\} \subset \{\phi,\{\phi\}\}$ (f) $\{\{\phi\}\} \subset \{\{\phi\},\{\phi\}\}$

3. For each of the following functions f: **Z**→**Z**, determine which one(s) is/are one-to-one and onto.
(a) $f(x) = x+7$ (b) $f(x) = 2x-3$ (c) $f(x) = -x+5$ (d) $f(x) = x^2$ (e) $f(x) = x^2 + x$ (f) $f(x) = x^3$

4. Consider the open statement $p(x, y)$: $y - x = y + x^2$, where the universe for each of the variables x, y comprises all integers. For each of the following statements, which one's/ones' truth value(s) is/are False?
(a) $p(1, 1)$ (b) $p(0, 1)$ (c) $\forall y\ p(0, y)$ (d) $\exists y\ p(1, y)$ (e) $\forall x \exists y\ p(x, y)$ (f) $\exists y \forall x\ p(x, y)$

5. Two hundred coins numbered 1 to 200 are put in a row across the top of a cafeteria table. Two hundred students are assigned numbers (from 1 to 200) and are asked to turn over certain coins. The student assigned number 1 is supposed to turn over all the coins. The student assigned number 2 is supposed to turn over every other coin, starting with the second coin. In general, the student assigned the number n, for each $1 \leqq n \leqq 200$, is supposed to turn over every nth coin, starting with the nth coin. How many times will the 200th coin be turned over?
(a) 14 (b) 15 (c) 16 (d) 17 (e) 18 (f) 19

6. Let A = {v, w, x, y, z}. Determine the number of relations on A that are
(a) reflexive and symmetric=1024 (b) equivalence relations=52 (c) reflexive and symmetric but not transitive=972 (d) equivalence relations that determine exactly two equivalence classes=15 (e) equivalence relations where $w \in [x]$ =15 (f) equivalence relations where $v,\ w \in [x]$ =5

7. Let $M = (S, I, O, \nu, \omega)$ be a finite state machine with $I=O=\{0, 1\}$ and S, ν, and ω determined by the state diagram shown in Fig. 1. Starting in state s_0, if the output for an input string x is 0000001, determine all possibilities for x.
(a) 1111110 (b) 1111011 (c) 1111111 (d) 1011110 (e) 1100110 (f) 1010110

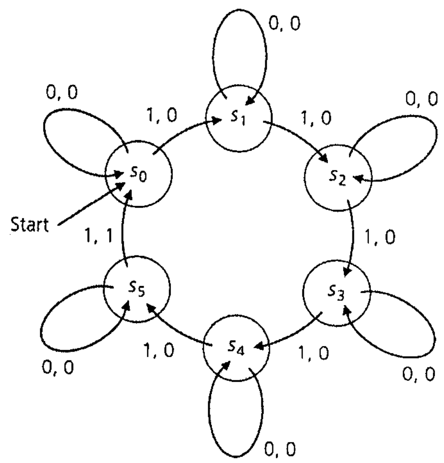

Fig. 1

8. Let R be a binary relation on the set of all positive integers such that $R = \{(a,b) | a = b^2\}$. R is
(a) reflexive (b) symmetric (c) asti-symmetric (d) transitive (e) an equivalence relation (f) a partial ordering relation

注意：背面有試題

國立中央大學95學年度碩士班考試入學試題卷

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二、非選擇題（60%）

1. The four input lines for the network in Fig. 2 provide the binary equivalents of the numbers 0, 1, 2, . . . , 15, where each number is represented as *abce*, with *e* the least significant bit. Draw the two-level gating network for *g* as a minimal sum of products. (5%)

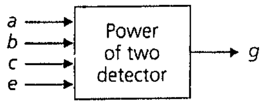

Fig. 2

2. Given 8 Perl books, 17 Visual BASIC† books, 6 Java books, 12 SQL books, and 20 C++ books, how many of these books must we select to insure that we have10 books dealing with the same computer language? (5%)

3. Solve the following recurrence relation. (10%)

   $a_n = -2a_{n-2} - a_{n-4}$, $a_0 = 0$, $a_1 = 1$, $a_2 = 2$, $a_3 = 3$

4. Solve the following simultaneous recurrence relations. (10%)

$$a_{n+1} = 2a_n + b_n + c_n,\ n \ge 0$$
$$b_{n+1} = b_n - c_n + 4^n,\ n \ge 0$$
$$c_{n+1} = c_n - b_n + 4^n,\ n \ge 0$$
$$a_0 = 1,\ b_0 = c_0 = 0$$

5. Let *G* be a graph with at least two vertices. Prove that *G* has two vertices with the same degree. (10%)

6. Let *G* be a connected planar graph without self-loops. Prove that *G* has a vertex with degree smaller than 6. (10%)

7. Find the maximum heights of the following trees. (10%)

   (a) A binary tree with *n* vertices.

   (b) A complete binary tree with *n* vertices.