

國立中央大學八十六學年度碩士班研究生入學試題卷

所別： 產業經濟研究所 乙組 科目： 乙統計學 共 2 頁 第 1 頁

Statistics 50%

Complete the following statements. Each blank may contain more than one word. (Each blank weights equally.)

1. Let X_1, X_2, X_3 denote a random sample of size $n=3$ from a distribution with the probability density function $f(x) = \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{x-1}$, $x=1,2,3,\dots$

(1) The population distribution has a _____ distribution.

(2) $P(X_1=1, X_2=3, X_3=1) =$ _____

(3) If Y equals the maximum of X_1, X_2, X_3 , then $P(Y \leq 2) =$ _____



2. Let X_1, X_2, \dots, X_{20} denote the outcomes on Bernoulli trials with the probability of success being 0.4. Define $Y = \sum_{i=1}^{20} X_i$.

(1) Y has a _____ distribution with expectation being _____ and variance being _____.

(2) Y can be appropriately approximated by a _____ distribution, so $P(Y \leq 2)$ approximately equals to _____.

3. Let X_1, X_2, \dots, X_8 be a random sample of size $n=8$ from a Poisson distribution with mean λ . Reject the simple null hypothesis $H_0: \lambda = 0.5$ and accept $H_1: \lambda > 0.5$ if the observed sum $\sum_{i=1}^8 X_i \geq 8$.

(1) Define $Y = \sum_{i=1}^8 X_i$. Then Y has a _____ distribution with mean _____.

(2) The significance level of the test is _____.

(3) The power function $K(\lambda)$ of the test is _____.

4. The _____ is the set of all possible outcomes of an experiment, while a _____ is a function that assigns to each outcome of that experiment one and only one real number.

5. Let X be $(\mu, 100)$. To test $H_0: \mu = 80$ against $H_1: \mu > 80$, let the critical region be defined by $C = (\bar{x} \geq 83)$, where \bar{x} is the sample mean of a random sample of size $n=25$ from this distribution. The significance level of this test α equals _____ and the P-values corresponding to $\bar{x}=82.5$ is _____. ($P(0 \leq z \leq 1.25) = 0.3944$, $P(0 \leq z \leq 1.50) = 0.4322$)

6. Let X_1, X_2, X_3, X_4 equal the cholesterol level of a women under the age of 50, a man under 50, a woman 50 or older, and a man 50 or older, respectively. Assume that the distribution of X_i is $N(\mu_i, \sigma^2)$, $i=1,2,3,4$. We shall test the null hypotheses $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ using 7 observations of each X_i . The critical region for an $\alpha = 0.05$ significance level is $F_{0.05}(v_1, v_2) = 3.01$, where v_1 and v_2 represent the related degrees of freedom in this case. The corresponding ANOVA summary table is shown below.

| Source | SS | DF | MS | F |
|-----------|-----------|----|----|---|
| Treatment | 12,280.86 | | | |
| Error | | | | |
| Total | 40,715.43 | | | |

Note: $F_{0.10}(v_1, v_2) = 2.33$, $F_{0.05}(v_1, v_2) = 3.01$, $F_{0.025}(v_1, v_2) = 3.72$, $F_{0.010}(v_1, v_2) = 4.72$

Then the corresponding p-value of the test is _____, and the F-value of the test is _____.

7. A package, say A, of 24 crocus bulbs contains 8 yellow, 8 white, and 8 purple crocus bulbs. A package, say B, of 24 crocus bulbs contains 6 yellow, 6 white, and 12 purple crocus bulbs. One of the two packages is selected at random. If 3 bulbs from this package were planted and all 3 yielded purple flowers, the conditional probability that package B was selected is _____.

8. Suppose $P(A)=0.5$, $P(A \cup B)=0.6$. $P(B)=$ _____ if A is independent of B.

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Econometrics 50%

9. (10%) Determine whether the following models are linear regression models?

(a) $Y_i = \beta_0 + \beta_1 \left(\frac{1}{X_i} \right) + \beta_2 X_{2i} + u_i$

(b) $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_1 \beta_2 X_{2i} + u_i$

(c) $Y_i = e^{\beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i}$

(d) $Y_i = \frac{1}{\beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i}} + u_i$

(e) $Y_i = \beta_0 X_{1i}^{\beta_1} X_{2i}^{\beta_2} e^{u_i}$

10. (16%) Suppose you are asked to estimate a linear regression model

$$Y_i = \beta_0 + \beta_1 X_{1i} + u_i$$

based on the following data:

$$\sum_{i=1}^{10} x_i y_i = 16,800 \quad \sum_{i=1}^{10} x_i^2 = 33,000 \quad \sum_{i=1}^{10} y_i^2 = 8,890$$

$$\sum_{i=1}^{10} X_i = 170 \quad \sum_{i=1}^{10} Y_i = 1,110$$

where y_i and x_i are deviations from the mean values of Y and X .

- What are the values of β_0 and β_1 ?
- What are the values of the variance and standard error of β_1 ?
- What is the value of the coefficient of determination?
- What is the F value?

11. (12%) Suppose you are asked to estimate a regression model

$$Y_i = \beta_0 + \beta_1 X_{1i} + u_i$$

using the method of maximum likelihood. Please derive the estimators of β_0 , β_1 and σ^2 . Is the estimator of σ^2 biased?

12. (12%) Suppose you had data to consider the Cobb-Douglas production function

where Y = output, L = labor input, and K = capital input.

- How would you test the hypothesis that there are constant

$$Y_i = AL_i^\alpha K_i^\beta e^{u_i}$$

returns to scale based on the unrestricted and restricted least squares?

- If the data covers two periods, 1951-1970 and 1971-1990, how would you test that the production function has undergone a structural change between the two periods.