

國立中央大學八十六學年度碩士班研究生入學試題卷

所別: 產業經濟研究所 甲組 科目: 基礎數學 共 / 頁 第 / 頁



1. If $3x^2z + y^3 - xyz^3 = 0$, find $\frac{\partial z}{\partial x}$ (12%)

2. Find $\int_0^1 \int_0^{\sqrt{1-x^2}} (4-x^2-y^2)^{-\frac{1}{2}} dy dx$. (12%)

3. Find the maximum and minimum values of $f(x,y,z) = x + 2y + 3z$ on the ellipse that is the intersection of the cylinder $x^2 + y^2 = 2$ and the plane $y + z = 1$. (15%)

4. Let $f(x) = \frac{3}{5}x^{\frac{5}{3}} - 3x^{\frac{2}{3}}$
 (a). Indicate where f is increasing and where it is concave down. (10%)
 (b). Sketch the graph of the function $f(x)$. (6%)

5. Find the interval of convergence of $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2 3^k} (x+2)^k$ (15%)

6. Find the gradient vector of the ellipsoid $9x^2 + 4y^2 + 9z^2 = 34$ at the point $P(1,2,-1)$. Write the equation of the tangent plane to this surface at the given point. (10%)

7. Let $T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be a linear transformation from \mathbb{R}^3 into \mathbb{R}^3 .

- (a). Find the null space of T .
 (b). Find T^{-1} and its matrix representation. (10%)

8. Solve $y'' - 2y' - 3y = 8e^{3x} + \cos 2x$. (10%)