

# 國立中央大學八十七學年度碩士班研究生入學試題卷

所別: 產業經濟研究所 甲組 科目: 甲統計學 共 1 頁 第 1 頁

1.(30%) Let random variable  $X$  has a density function  $f(x)$ , cumulative distribution function  $F(x)$ , mean  $\mu$ , and variance  $\sigma^2$ . Define  $Y=c+dX$ , where  $c$  and  $d$  are constants satisfying  $-\infty < c < \infty$  and  $d > 0$ .

- (a) Select  $c$  and  $d$  so that  $Y$  has mean 0 and variance 1.
- (b) What is the correlation coefficient between  $X$  and  $Y$ ?
- (c) If  $X$  is symmetrically distributed about  $\mu$ , is  $Y$  necessarily symmetrically distributed about its mean?

2.(20%) Kitty Oil Co. has decided to drill for oil in 10 different locations; the cost of drilling at each location is \$10,000 (i.e. total cost is \$10,000). The probability of finding oil in a given location is only 1/5. If oil is found at a given location, the amount of money the company will get selling oil (excluding the initial \$10,000 drilling cost) from that location is an exponential random variable with mean \$50,000.

- (a) What is the expected profit of this company?
- (b) What is the probability that the Kitty Oil Co. could obtain more than \$10,000 profits, if the oil is found exactly in two locations.

3.(30%) Let  $X_1$  and  $X_2$  be a random sample of size 2 from the uniform distribution  $U(0,1)$ . And  $Y=X_1+X_2$ .

- (a) What is the joint probability density function of  $X_1$  and  $X_2$ ?
- (b) What is the probability density function of  $Y$ ? Please graph it.
- (c) What is the smallest sample size needed for  $Y$  to be approximated by a normal distribution? Why?

4. (10%) Cars arrive at a toll booth at a mean rate of five cars every 10 minutes according to a Poisson process. Define  $X$  as the time that the toll collector has to wait before collecting the eighth toll.

- (a) What is  $X$  distributed? What is the expected waiting time?
- (b) Find the point at which the waiting time obtains its maximum.

5.(10%) What do the assumptions of the classical regression model really mean? What does a linear regression model mean? Under what circumstances could a non-linear model be transformed to linear? Please give an example.

