

所別：產業經濟研究所碩士班 甲組 科目：微積分

可選擇以英文或中文作答。

1. (25%) Prove "If $f(x)$ is a concave function, then for any constant k , the set

$S = \{x \mid f(x) \geq k\}$ is convex; but the converse is not true."

2. (20%) Evaluate $\int_{-1}^1 \frac{1}{x^3} dx$.

3. (30%) Derive the general solution for the following model of differential equations.

$$\begin{aligned}z &= \frac{1}{6} - 3x + y, \\ \frac{dx}{dt} &= -\frac{1}{2}(m - z), \\ \frac{dy}{dt} &= \frac{3}{4}(z - y),\end{aligned}$$

where m is an exogeneous variable.

4. (25%) Consider the CES production function, $Q = A[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-1/\rho}$,

where $A > 0$, $0 < \delta < 1$, $-1 < \rho \neq 0$. Given constant input prices r and w , show

that the locus of points (K^*, L^*) that minimize the cost, $C = rK + wL$ at various

levels of Q (i.e., expansion path) is a straight line emanating from the point of

origin in the (K, L) space.

參考用