國立中央大學101學年度碩士班考試入學試題卷

所別：財務金融學系碩士班 乙組(一般生)　科目：微積分　共 1 頁 第 1 頁

本科考試禁用計算器　*請在試卷答案卷（卡）內作答

(20%) 1. Define a bivariate function of x and y as follows:

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} e^{\frac{-1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right)+\left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]}$$

$$= \left[\frac{1}{\sigma_x\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2}\right]\left[\frac{1}{\sigma_y\sqrt{2\pi}\sqrt{1-\rho^2}} e^{\frac{-1}{2}\left(\frac{y-\alpha}{\sigma_y\sqrt{1-\rho^2}}\right)^2}\right],\ \alpha = \mu_y + \rho\frac{\sigma_y}{\sigma_x}(x-\mu_x)$$

where $-\infty < x, y < \infty$, $-1 < \rho < 1$, $\mu_x \in \mathcal{R}, \mu_y \in \mathcal{R}$, $\sigma_x \in \mathcal{R}^+$ and $\sigma_y \in \mathcal{R}^+$. Please compute the following integration.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{ax+by} f(x,y)\,dx\,dy,$$

where a and b are two arbitrary real values. (Hint: $\int_{-\infty}^{\infty} e^{cw}\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{w-\mu}{\sigma}\right)^2} dw = e^{c\mu+\frac{1}{2}c^2\sigma^2}$).

(20%) 2. Please use Taylor's formula for $f(x,y) = e^x \ln(1+y)$ at the origin to find the quadratic approximation of f near the origin.

(20%) 3. Please compute $\frac{\partial P(x)}{\partial x}$ and $\frac{\partial^2 P(x)}{\partial x^2}$, where $P(x)$ is defined as follows:

$$P(x) = Ke^{-rT}\Phi(-d_2) - x\Phi(-d_1),$$

where $d_1 = \frac{\ln\left(\frac{x}{K} + \left(r+\frac{1}{2}\sigma^2\right)T\right)}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$, $\Phi(a) = \int_{-\infty}^{a}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}dz$, and K, r, T and σ are constant.

(20%) 4. Let $f(t)$ be a continuous function defined for $0 \le t \le T$ and suppose that f has a continuous derivatives. Please compute the following limit:

$$\lim_{\|\Pi\|\to 0}\sum_{j=0}^{n-1}\left[f(t_{j+1}) - f(t_j)\right]^2,$$

where $\Pi = \{t_0, t_1, ..., t_n\}$ with $0 = t_0 < t_1 < t_2 < ... < t_n = T$ and $\|\Pi\| = \underset{j=0,1,\ldots,n-1}{\text{Max}}(t_{j+1} - t_j)$.

(20%) 5. If $f(x) = \frac{1}{2}$, $-1 < x < 1$, zero elsewhere, is the probability density function of the random variable X, find the probability density function of $Y = X^2$.