國立中央大學105學年度碩士班考試入學試題

所別： 財務金融學系 碩士班 甲組(一般生)　　共3頁 第1頁
財務金融學系 碩士班 乙組(一般生)

科目： 統計

本科考試禁用計算器　　＊請在答案卷(卡)內作答

問答題

答題請依照題目順序、標示題號、簡捷作答每題皆須提示理由解釋、計算過程或證明，否則不予計分。

1. 話說寧真心 與虛太宇 相約在某日晚上 6:00–7:00 在“真情永遠”演唱會的入口相見，彼此約定先到的等對方 15 分鐘為限，相見就一起入場；如果等超過 15 分鐘就直接散場。如果他倆分別都可以在 6:00–7:00 任何時刻抵達演唱會入口，試算兩人手牽手一起入場聽演唱會的機率。(5%)

2. 某坊間分析師宣稱，利用跨國性的投資組合佈局財富管理，才能有效降低國內藍綠惡鬥所致的經濟與系統性風險。為了協助客戶財富管理選擇投資標的，他針對全球涵蓋 50 國的股市加權股價指數去算平均值與標準差後，得到各國指數的“Sharpe ratio”推薦客戶投資其中前五名。請以統計學中的相關概念評述該分析師的作法與推薦名單的參考性適不適合推廣？請說明理由！(10%)

3. 凌波微步 (random walk process，有人也稱作隨機漫步) 是財金產業常拿來刻劃金融資產價格一個很重要的隨機過程。請試說明何謂“凌波微步”(5%)，並以任何一種你熟悉的統計軟體語法，說明你如何模擬出一個樣本數為 500 的凌波微步。(10%)

4. 金融機構通常會被金管當局要求利用不同的模型或方法去設算風險值 (Value-at-Risk)，做為控管極端風險與風險性資本計提準備的參考。假設蘑根絲彈力銀行習慣性委託發奇銀行幫忙設算在 95% 信心水準下，未來百日每天可能面對的 VaR (單日最大可能損失值)。如果有一天實際銀行的部位損失超出此 VaR 部位，我們稱之為觸發穿透 (violation or exceedance)。根據最新接獲的通報發現，在過去這一百天中，蘑根絲彈力銀行有 15 天觸發穿透所提列 VaR 損失。若你想評估發奇銀行所幫忙設算的風險值是否有問題，嘗試定義一個隨機變數，並利用你的統計知識回答下列問題:

   (a) 請寫下您認為合理的檢定假說以及其中涉及的假設為何。(5%)

   (b) 請問在發奇銀行所設算的 95% VaR 下，百日內出現 8 天觸發穿透的機率為何？(5%)

   (c) 請以適當地推論與檢定說明蘑根絲彈力銀行該不該繼續跟發奇銀行續約？為什麼？(5%)

   (d) 如果冰雪聰明的你發現這觸發穿透的幾天都集中在某個月，你會如何建議發奇銀行改善他們的作業？(5%)

注意:背面有試題

5. Consider an linear ordinary least-squares (OLS) model between Y and X as

$$Y_i=\alpha+\beta X_i+\varepsilon_i,\ i=1,\ldots,N.$$

Assume the residual terms ε_i are independently and identically distributed with zero mean and variance σ^2, that is, ε_i are $iid(0, \sigma^2)$.

(a) Write down the detailed statistical definitions for the term "$iid(0, \sigma^2)$". (5%)

(b) Write down the Normal Equations. (5%)

(c) Derive the least squares (LS) estimators of α and β. (5%)

(d) Derive the *estimated* standard errors of the LS estimators of α and β. (5%)

(e) Suppose now the residuals have a positive mean, $E(\varepsilon_i)=c>0$. Is the LS estimator of β unbiased? (5%)

6. We aim to test if stock investors, in general, benefit from large R&D investments. To test our idea, in the beginning of a given year, we draw a sample of 100 firms that have undertaken large R&D investment projects (Sample 1), and another sample of 200 firms that have not (Sample 2). We then trace the stock returns of these 300 firms for one year. Suppose over this one year, in Sample 1 there are 80 firms have positive returns and in Sample 2 there are 120 firms have positive returns. Please perform the following tests. *Be sure to write down the tested hypothesis.*

(a) Test at 10% significance level that the undertaking of large R&D investments has no effect on firms' likelihood of experiencing positive stock returns. (5%)

(b) Suppose the average returns of Sample 1 and Sample 2 are 6% and 3%, and the standard deviations of the returns of Sample 1 and Sample 2 are 12% and 16%, respectively. Test the hypothesis at 10% significance level that the undertaking of large R&D investments has no effect on the level of subsequent returns, assuming unequal variances. (5%)

注意:背面有試題

7. Assume we have a relationship between variables y and x as follows:

$$y = \beta x + e,$$

where $e \sim N(0, \sigma_e^2)$. However, sometimes we are not able to observe the "actual" values of x or y in data, but can only observe the value of variables $\tilde{x}$ and $\tilde{y}$ that are related to x and y in the following manner:

$$\tilde{x} = x + u, \text{ where } u \sim N(\sigma_u^2),$$
$$\tilde{y} = y + v, \text{ where } v \sim N(\sigma_v^2).$$

(a) When we observe x but not y, and can only estimate the model $\tilde{y} = \beta x + \varepsilon$, is the LS estimator $\hat{\beta}$ biased, and why? (3%)

(b) When we observe y but not x, and can only estimate the model $y = \beta \tilde{x} + \varepsilon$, is the LS estimator $\hat{\beta}$ biased, and why? (4%)

8. We believe that firm characteristics F can predict subsequent return R, and thus draw a sample S and estimate the model $R_i = \alpha + \beta F_i + e_i$, where $e_i \sim N(0, \sigma^2)$ and $i=1,\ldots,N$. Suppose there is *new* observation, firm j that is not in the sample S, and its value of variable F is F_0 ($F_j=F_0$). Using the estimated model, we may derive the predicted return for firm j as $\hat{R}_0 = \hat{\alpha} + \hat{\beta} F_0$. Suppose the actual return for firm j is R_0, what is the variance of prediction error $R_0 - \hat{R}_0$? (Hint: We know $E[(\hat{\beta} - \beta e=0)$ (8%)