

國立中央大學 105 學年度碩士班考試入學試題

所別： 財務金融學系 碩士班 乙組(一般生)

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科目： 微積分

本科考試禁用計算器

*請在答案卷(卡)內作答

(10%) 1. Please compute

$$\int_0^a \int_0^b e^{\max(b^2x^2, a^2y^2)} dy dx$$

where a and b are positive numbers.

(10%) 2. Please use the total differential method to approximate the value of $\sqrt{2.98^2 + 4.01^2}$.

(10%) 3. Please evaluate $\int e^{-x^2} dx$ as an infinite series.

(10%) 4. Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{n^n}{n!}$$

(10%) 5. Test the convergence of the series

$$\sum_{n=1}^{\infty} \left(\frac{2n+3}{3n+2} \right)^n$$

(10%) 6. Please compute $\iint_D (x+2y) dx dy$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.

(10%) 7. Please compute

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 (x^2 + y^2) dz dy dx.$$

(10%) 8. Please use the spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.

(10%) 9. Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.

(10%) 10. Let $Z = X/Y$. Find the probability density function of Z if X and Y are independent and both exponentially distributed with mean one.