須有計算過程

- 1. (10%) Please solve the general solution for the following ordinary differential equation:

$$x\frac{dy}{dx} + (1+x)y = e^x$$

2. (10%) Please calculate the following improper integral:

$$\int \frac{3e^x}{1+e^{0.5x}}dx$$

3. (10%) Please calculate the following improper integral:

$$\int \frac{dx}{x^2\sqrt{x^2-4}}$$

4. (10%) Find the expected value, variance, and standard deviation of the random variable X associated with the probability density function.

$$f(x) = \frac{32}{15x^3}$$

on $[1,4]$

5. (10%) Find the sum of the geometric series if it converges

$$\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1}{\sqrt{2}}\right)^n$$

6. (10%) Find the area of the region enclosed by the parabola $y = 2-x^2$ and the line $y = -x$

7. (10%) Find the limit (a) $\lim_{x\to(\pi/2)^-}\frac{\sec x}{1+\tan x}$ (b) $\lim_{x\to\infty}\frac{\ln x}{2\sqrt{x}}$

8. (10%) Evaluate the following improper integral $\int_1^{\infty}\int_{e^{-x}}^1\left(\frac{1}{x^3y}\right)dydx$

9. (10%) Evaluate

$$\int_0^a\int_0^b e^{\max\left(b^2x^2,a^2y^2\right)}dydx$$

where a and b are positive numbers and

$$\max\left(b^2x^2,a^2y^2\right) = \begin{cases} b^2x^2 & \text{if} \quad b^2x^2 \geq a^2y^2 \\ a^2y^2 & \text{if} \quad b^2x^2 < a^2y^2 \end{cases}$$

10. (10%) Define the gamma function $\Gamma(x) = \int_0^{\infty}t^{x-1}e^{-t}dt$, please evaluate $\Gamma(1/2)$