

國立中央大學 110 學年度碩士班考試入學試題

所別： 財務金融學系 碩士班 乙組(一般生)

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科目： 微積分

本科考試禁用計算器

*請在答案卷(卡)內作答

Instruction: This is a closed book exam. There are 10 problems and each is worth 10 points. Write everything CLEARLY, any ill-explanation or bad handwriting on your answer sheet should cause no credits. It is YOUR responsibility to turn in a clean, well-organized answer sheet.

Problem (1) Find

$$\int \exp(ax) \cos(bx) dx.$$

Problem (2) Derive a formula for the volume of a right circular cone of radius r and height h .

Problem (3) Find the relative extrema of the function

$$f(x, y) = x^3 - 12xy + 8y^3, (x, y) \in \mathbb{R}^2.$$

Problem (4) Find the interval of convergence of the following series:

$$f(x) := \sum_{n=1}^{\infty} \frac{x^n}{2+n^2}.$$

Problem (5) Find the particular solution of

$$x^4 y' - 9y' - x^3 y + 3xy = 0, \quad y(1) = 16.$$

Problem (6) Let

$$f(x) = \frac{4x}{x^2 + 3}.$$

It is clear that 0 is a root of $f(x) = 0$. Use Newton's method with initial $x_0 = 1$ to obtain a sequence of "approximations". Does the above "approximations" approach 0? If not, give the reason.

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Problem (7) Let $f(x) = x^3 - x$ and $g(x) = x - x^2$. Find the area of the region enclosed by the graphs of $y = f(x)$ and $y = g(x)$.

Problem (8) Show that

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$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 1.$$

You can prove this by first showing that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dx dy = 1.$$

Problem (9) Use the method of Lagrange multiplier to find the minimum of the function

$$f(x, y, z) = xy + xz + yz,$$

subject to the constraint $xyz = 1000$.

Problem (10) Find

$$\lim_{x \rightarrow 0} x^x,$$

if it exists.