

# 國立中央大學 113 學年度碩士班考試入學試題

所別： 財務金融學系 碩士班 乙組(一般生)

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科目： 微積分

\*本科考試禁用計算器

## 計算題

計算題應詳列計算過程，無計算過程者不予計分

(10%) 1. Based on the change from Cartesian coordinate to polar coordinate, please show that

$$\int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}x^2\right) dx = \sqrt{2\pi}.$$

(10%) 2. The joint pdf of X and Y is given as follows:

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{(x^2 - 2\rho xy + y^2)}{2(1-\rho^2)}\right), \quad -\infty < x, y < \infty.$$

Please derive the marginal pdf of X by integrating  $f_{X,Y}(x,y)$  over y.

(10%) 3. Find the area of the region bounded by the graphs of  $f(x) = x^2 + x + 4$  and  $g(x) = 3x + 7$ .

(10%) 4. The region bounded by the curve  $y = x^2 + 7$  and the line  $y = 2x + 10$  is revolved about x-axis to generate a solid. Find the volume of the solid.

(20%) 5. Find the volume of the region bounded by the surfaces  $z = x^2 + y^2$  and  $z = 8 - x^2 - y^2$ .

(20%) 6. Use the transformation  $x = u + (1/2)v$  and  $y = v$  to evaluate the integral

$$\int_0^2 \int_{y/2}^{(y+4)/2} y^3(2x-y) \exp((2x-y)^2) dx dy$$

by first writing it as an integral over a region in the  $uv$ -plane.

(20%) 7. Find the absolute maxima and minima of the function on the given domain.

$$f(x,y) = x^2 + xy + y^2 - 6x + 2, \quad 0 \leq x \leq 5, \quad -3 \leq y \leq 0.$$