國立中央大學八十六學年度碩士班研究生入學試題卷

所別: 財務管理研究所 甲.乙丙丁組　科目: 統計學　共 一 頁 第 1 頁

Instructions: Answer the following questions. Make and state your own assumptions for questions where the information is not sufficient for you to solve them. For example, if you need the corresponding p-value of a normally distributed random variable evaluated at 2.5, you may indicate the value as, say, $Pr(x \geq 2.5)$, where $x \sim \mathcal{N}(0,1)$.

PART I

1. (25 points) Given $f(x|\theta) = 1 + \theta^2[x - \frac{1}{2}]$, $0 \leq x \leq 1$, $0 \leq \theta \leq \sqrt{2}$,
   (a) (10 points) Calculate the mean and variance of x?
   (b) (15 points) If you are given only one observation X, how will you test the null hypothesis $H_0 : \theta = 0$ against the alternative hypothesis $H_1 : \theta > 0$? Write down the distribution of the statistic you use. Derive the "critical region" for this test, given the type I error of 10%?

2. (25 points) Suppose you are given a sample of observations which are independently and identically distributed, i.e., $y_1, \ldots, y_n \overset{iid}{\sim} \mathcal{N}(\mu, \sigma^2)$. The sample mean and variance are: $\bar{y} = 0.0035, \hat{\sigma}^2 = 0.025$. Now two students would like to make some statistical inference about the sampling distribution of the sample mean, but they do not agree with each other.
   (a) Student A says that the sampling distribution of $\bar{y}$ is normal with mean 0.035 and variance 0.025, i.e., $\bar{y} \sim \mathcal{N}(0.035, 0.025)$.
   (b) Student B does not agree with Student A. She thinks that one has to test if the mean is significantly different from zero first. So she tests the null hypothesis $H_0 : \bar{y} = 0$, and could not reject it. Therefore she concludes the sampling distribution should be: $\bar{y} \sim \mathcal{N}(0, 0.025)$.

   Who is correct? Are there any problems with their statements?

PART II

3. (1)(5 points)

   Consider the least squares residuals, given by $y_i - \hat{y}_i$ $(i=1,2,\ldots,n)$.

   Show that $\sum_{i=1}^{n} \frac{\hat{y}_i}{n} = \bar{y}$

   (2)(5 points)

   Show that, for the simple linear regression model,

   $$\sum_{i=1}^{n} (y_i - \hat{y}_i) = 0$$

   (3)(7 points)

   The estimator of the error variance, σ^2, is given by

   $$S^2 = \sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)^2}{(n-2)}$$

   show that the estimator is unbiased, prove $E(S^2) = \sigma^2$.

   (4)(8 points)

   For the simple linear regression model, show that

   $$b_1 = \frac{S_{xy}}{S_{xx}} \quad \text{and} \quad \bar{y} = \sum_{i=1}^{n} \frac{y_i}{n}$$

   have zero covariance.

4. (1)(8 points)

   Let X be a random variable having an exponential density with parameter λ.

   Find the density of $Y = X^{1/\beta}$, where $\beta \neq 0$.

   (2)(8 points)

   Let X and Y be independent random variables each having an exponential distribution with parameter λ. Find the distribution of X+Y.

   (3)(9 points)

   Let X and Y be independent and uniformly distributed over (0,1). Find the density of X+Y.