國立中央大學八十六學年度碩士班研究生入學試題卷

所別: 財務管理研究所 乙,丁組 科目: 微積分 共 1 頁 第 1 頁

1. Evaluate (14%)

(a) $\lim_{h\to 0} \dfrac{\frac{1}{8+3h} - \frac{1}{8}}{h}$

(b) $\lim_{x\to 0} \dfrac{\tan x}{x}$

2. Find the derivatives $f'(x)$ of the following functions. (28%)

(a) $f(x) = \dfrac{x}{\sin x}$ (b) $f(x) = \tan x - x \ln x$

(c) $f(x) = \sin(\sin x)$ (d) $f(x) = 2^{\ln x}$

3. Find the following integrals (28%)

(a) $\int (\sec 6x)^2 \, dx$ (b) $\int x \ln x \, dx$

(c) $\int \dfrac{4}{x^2-1} \, dx$ (d) $\int_0^3 \sqrt{9-x^2} \, dx$

4. Find all points on the curve with equation $x^2 - xy + y^2 = 3$ (10%) at which the tangent line is horizontal.

5. Test for convergence for the following series. (10%)

(a) $\sum_{n=0}^{\infty} \dfrac{2^n}{n!}$ (b) $\sum_{n=2}^{\infty} \dfrac{\log_n (n!)}{n^3}$

6. Prove that no straight line can be tangent to the (10%) graph of $y = x^2$ at two different points.