

所別：財務金融學系碩士班 乙組科目：微積分

1. (20%) Find the following limit

(a). (8%)

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x)$$

(b). (6%)

$$\lim_{x \rightarrow \infty} \left(1 - \frac{n}{x}\right)^{-x}$$

where  $n$  is a constant.

(c). (8%)

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots - \frac{1}{2x}\right)$$

2. (10%) Examine the following series for convergence or divergence with proof.

(a). (5%)

$$\sum_{n=1}^{\infty} n^3 e^{-n}$$

(b). (5%)

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n+1}{n} \left(\frac{1}{2}\right)^n$$

3. (15%) Evaluate

(a). (8%)

$$\int_2^4 \frac{x^2 + 1}{(2x - 3)^2} dx$$

(b). (7%)

$$\int_4^9 \frac{\sqrt{x}}{1 + x\sqrt{x}} dx$$

4. (20%) Please compute

(a). (10%)

$$\int_0^1 \left( \int_0^1 \frac{x-y}{(x+y)^3} dy \right) dx$$

(b). (10%)

$$\int x (\ln x)^2 dx$$

5. (15%) Use the Maclaurin series for  $e^x$  to estimate  $\int_0^1 e^{-x^2} dx$  to second-decimal place accuracy.

6. (20%) Given that

$$\sigma(t, T) = \sigma(t, t) e^{-\int_t^T k(x) dx}$$

and define

$$y(t) = \int_0^t \sigma^2(s, t) ds$$

please find what  $dy(t)$  is?

