所別：財務金融學系碩士班 甲組 科目：統計

乙組．丙組

1. (15%) Mr. and Mrs. Strong are very keen on buying lottery as they believe they are a lucky couple. However, they have never chosen and will not choose the same numbers. As you may have already known, this lottery is to draw 6 balls from a set of 49 balls, numbered from 1 to 49.
   (1) (2%) How many possible outcomes are there in the sample space?
   (2) (3%) What is the chance with which Mr. Strong chooses none of the numbers drawn?
   (3) (5%) If Mrs. Strong gets all the numbers wrong, then how will Mr. Strong's prospects (the probabilities of getting all wrong and all correct) change?
   (4) (5%) Show that the expected value of the correct number of choices made by Mr. Strong is less than 1.

2. (15%) 最近台中開了一家 LV 名品旗艦店，疼老婆的羅冰先生也準備了好幾張信用卡帶著老婆來朝聖。羅冰進到店裡五分鐘後心想「天啊！太貴了」，雖然羅冰很疼愛老婆，但還是擔心老婆會過度消費他的疼愛，於是準備在老婆瘋狂 Shopping 之前給他教育一翻。他說:「我親愛的老婆啊！你知道嗎？像你氣質這麼高尚的人是不需要靠 LV 來襯托的。根據 NCU 資訊公司的調查，台灣總人口中氣質高尚與否的比率大約 4：6，氣質高尚的人當中會買 LV 的只有 30%，氣質不高尚的人當中會買 LV 的卻高達 80%，你如果可以回答出下面幾個問題的話，你就會了解像你這麼氣質出眾的人真的不需要買 LV 啊！當然，如果這些問體你都答得出來，那表示你真的是冰雪聰明，那就讓你隨便買吧！」這時羅冰先生突然尿急去了，機靈的羅太太爲了過過像貴婦般瘋狂 Shopping 的滋味，於是偷偷打電話向你求救了。問題如下：
   (1) (10%) 若在 LV 店內的 50 個客人是台灣人口組成的完完全全縮小版，暨高尚又會買 LV 的有幾個？不高尚但會買 LV 又有幾個？
   (2) (5%) 在結帳的那位小姐（不是店員喔！）是假高尚的機率有多少？

3. (20%) NCU Ltd. has three products. Let X, Y and Z denote the annual sales (in million dollars) of the three products. These three random variables have the following distributions: X~N(30, 4^2), Y~N(40, 6^2) and Z=exp(X). The correlation coefficient between X and Y is 0.5.
   (1) (4%) Describe the probability distribution of X+Y.
   (2) (5%) Derive the probability density function of Z.
   (3) (5%) Find the solution for $E[e^{ux}]$, which applies for all real and complex numbers u.
   (4) (6%) Calculate the mean and the variance of Z.

   ※ $f_{normal}(x) = \dfrac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$.

4. (15%) Please choose the correct answer for each of the following questions:

   (1) (5%) The farther out in the tail of a distribution our critical value falls, the greater the risk of making
   (a) type I error
   (b) type II error
   (c) Both type I and II error
   (d) none of the above

   (2) (5%) Rather than a series of t tests, analysis of variance is used because
   (a) it holds type I error at a constant level
   (b) it increases type I error
   (c) it increases type II error
   (d) it makes a number of decisions, whereas a series of t tests makes a single overall decision

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(3) (5%) The two-way Chi-square might be used as a nonparametric alternative instead of ——— when comparing two groups.

(a) confidence intervals

(b) standard deviation

(c) t ratio

(d) analysis of variance

5. (15%) Let X be a random variable. Define $M_X(\theta) = E\left[e^{\theta X}\right]$ as the moment generating function (MGF) of X, where $f(x)$ is the probability density function of X. Define the cumulant generating function of X by the log of the moment generating function as

$$\psi(\theta) = \log E\left[e^{\theta X}\right] = \log M_X(\theta)$$

Now if we transform the density function of X, $f(x)$, into $\tilde{f}(x)$ by the following equation

$$\tilde{f}(x) = \frac{e^{\theta X} f(x)}{M_X(\theta)} = e^{\theta X - \psi(\theta)} f(x)$$

(1) (10%) Please find which of the following is the mean of X under the new probability distribution $\tilde{f}(x)$?

(a) $[M_X(\theta)]''$ $\quad (= \frac{d^2}{d\theta^2} M_X(\theta))$

(b) $\psi'(\theta)$ $\quad (= \frac{d}{d\theta}\psi(\theta))$

(c) $\psi''(\theta)$ $\quad (= \frac{d^2}{d\theta^2}\psi(\theta))$

(p.s. You have to prove it otherwise no points will be given.)

(2) (5%) For a normally distributed random variable X with mean μ, and variance σ^2, the moment generating function of X is $M_X(\theta) = e^{\mu\theta + \frac{\sigma^2\theta^2}{2}}$. Please find the mean of X under the new probability distribution $\tilde{f}(x)$.

6. (20%) Let $T_1, T_2, \cdots, T_n$ denote the arrival time of some event. We call the sequence (T_i) a Poisson process with intensity λ if the inter-arrival times $T_{i+1} - T_i$ are independent and exponentially distributed with parameter λ, i.e. $\text{Prob}(T_n - T_{n-1} > t) = e^{-\lambda t}$.

Equivalently, letting $N(t)$ count the number of event arrivals in the time interval $[0, t]$, we say that $N = (N(t))_{t \geq 0}$ is a Poisson process with intensity λ if the increments $N(t) - N(s)$ are independent and have a Poisson distribution with parameter $\lambda(t - s)$ for $s < t$.

(1) (10%) Please find out the probability that $N(t) - N(s) = k$.

(2) (10%) If we define the default time x of a company as the first jump time of the Poisson process N. Please find the probability distribution of $x \leq t$, i.e. $\text{Prob}(x \leq t)$.