國立中央大學98學年度碩士班考試入學試題卷

所別：財務金融學系碩士班 乙組　　科目：微積分　　共 1 頁　第 1 頁

*請在試卷答案卷（卡）內作答

1. (20%) Please find the following limit

   (a) (10%) $\lim_{x\to\infty}\frac{x^2}{e}\int_e^{x^2} e^{t^2-x^4}\,dt$

   (b) (10%) $\lim_{(x,y)\to(0,0)}\frac{\sqrt{1-\cos\left(x^2+y^2\right)}}{x^2+y^2}=?$

2. (10%) Let f be a twice differentiable function with $f(1)=2$, $f'(1)=0$ and

   $f''(x)>-2$ for 0<x<1. Prove that $f(0)>1$.

3. (15%) $\lim_{n\to\infty}\sum_{t=1}^{n}\frac{3}{2n+3t-2}=?$

4. (20%) Please evaluate the following integrals.

   (a) (10%) Giving $|x|<1$, find $\int\frac{1}{1-x^2}dx=?$

   (b) (10%) Find the following integral by reversing the order: $\int_0^1\int_x^1 e^{y^2}\,dy\,dx=?$

5. (15%) Let $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$, $(x>0)$.

   (1) (10%) Prove that $\Gamma(x+1)=x\Gamma(x)$

   (2) (5%) Show that $\Gamma\left(\frac{1}{2}\right)=2\int_0^\infty e^{-x^2}dx$

6. (10%) Solving the following ODE: $y'=\frac{e^x+\cos x}{2y}$, $y(0)=2$.

7. (10%) The call option price at current time t=0 is $C_0=S_0N(d_1)-Ke^{-rT}N(d_2)$,

   where $d_1=\frac{\ln\left(\frac{S_0}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$, and $d_2=d_1-\sigma\sqrt{T}=\frac{\ln\left(\frac{S_0}{K}\right)+\left(r-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$; r,

   σ, T and K are constant. The function $N(x)$ is the probability that a standard

   normal random valuable is less than x. That is, $N(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}dy$. Please

   prove that $\frac{\partial C_0}{\partial S_0}=N(d_1)$.