1. For two independent Poisson random variables $X_1 \sim \text{Poisson}(\lambda_1)$ and $X_2 \sim \text{Poisson}(\lambda_2)$, show that $X_1 + X_2 \sim \text{Poisson}(\lambda_1 + \lambda_2)$. (15 points)

2. Each summer, Tiger Woods play 100 rounds of golf on the same 18 hole golf course. The course has 5 short holes, 10 medium holes, and 3 long holes. Each time he plays a short hole. There is probability of 0.003 that Tiger hits a hole-in-one. On medium and long holes, the hole-in-one probabilities are 0.002 and 0.001, respectively. Assume that different holes are independent.

   Hint 1: Poisson approximation:

   If $Y_n \sim Bin(n, p_n = \frac{\lambda}{n})$, then $P(Y_n = k) \to \frac{e^{-\lambda}\lambda^k}{k!}, k = 0,1,\ldots$ as $n \to \infty$.

   Hint 2: Use the result from Problem 1.

   (i) What is the numerical probability that Tiger hits 3 or more hole-in-one next summer? (10 points)

   (ii) Given that Tiger hits exactly 5 hole-in-one next summer, what is the conditional probability that exactly 2 were on shorts holes? (10 points)

3. Consider two independent random variables $X_1 \sim \text{Gumbel}(0,\beta)$ and $X_2 \sim \text{Gumbel}(0,\beta)$. The CDF of $\text{Gumbel}(\mu,\beta)$ is defined as

$$F(x) = \exp(-e^{\frac{x-\mu}{\beta}})$$

   , where $-\infty < x < \infty$. Show that $\max(X_1, X_2) \sim \text{Gumbel}(\beta \ln 2, \beta)$. (15 points)

4. Suppose the probability of getting an "1" when tossing a particular die is p. We want to use the following two experiments to propose estimates of p. By making reasonable assumptions, find the *maximum likelihood estimates* of p based on these two different approaches.

   (i) Tossing this die 50 times, resulting 5 "1"s, 10 "2"s, 5 "3"s, 10 "4"s, 5 "5"s, and 15 "6"s. (10 pts)

   (ii) For each trial, we keep tossing this die until an "1" appearing. The number of tossings of 15 trials has been recorded as follows:

   3 1 7 5 14 12 4 8 9 6 6 11 10 17 7

   (For example, in the first trial, the results of the first two tossings are not "1" while the result of the third tossing is "1". Therefore this trial is stopped and the number of tossings in the first trial is 3.) (10 pts)

5. The following is the pitching record for Wei-Yin Chen's 16 wins in year 2014. There are exactly 8 wins in home games and the other 8 wins in guest games.

Date	Opp.	Score	Home/Guest	ER	K	ERA
15-Sep	TOR	W 5-2	Home	2	6	3.18
31-Aug	MIN	W 12-8	Home	4	7	5.40
1-Aug	SEA	W 2-1	Home	1	8	1.23
10-Jul	WAS	W 4-3	Home	3	6	4.76
3-Jul	TEX	W 5-2	Home	2	4	3.00
11-Jun	BOS	W 6-0	Home	0	7	0.00
9-May	HOU	W 4-3	Home	2	4	2.57
14-Apr	TB	W 7-1	Home	1	4	1.42
10-Sep	@BOS	W 10-6	Guest	1	4	1.29
20-Aug	@CWS	W 4-3	Guest	3	7	3.68
24-Jul	@SEA	W 4-0	Guest	0	3	0.00
19-Jul	@OAK	W 8-4	Guest	3	4	5.40
1-Jun	@HOU	W 9-4	Guest	1	6	1.69
15-May	@KC	W 2-1	Guest	1	1	1.69
21-Apr	@BOS	W 7-6	Guest	3	5	5.40
8-Apr	@NYY	W 14-5	Guest	4	3	7.20

參考用

(i) We want to know if there exists any significant difference between home games or guest games in Chen's win performance in **ERA**. We shall test the null hypothesis H_0: $\mu_h = \mu_g$, where μ_h represents the mean ERA in home win games, and μ_g represents the mean ERA in guest win games. Please perform an ***F*** test at $\alpha = 0.05$ and summarize the results in an ANOVA table. (10 pts)

(ii) We want to build a simple linear regression model on the relationship between Chen's **K** and **ER**. Let $\{x_i\}$'s be his Ks and $\{y_i\}$'s be his ERs in his 16 wins. Use the *least squares method* to find the estimated regression equation. Furthermore, compute the coefficient of determination. (10 pts)

(iii) Perform an ***F*** test on the hypothesis: H_0: $\beta_1 = 0$ at $\alpha = 0.05$ and summarize the results in an ANOVA table. (10 pts)

注意：背面有試題

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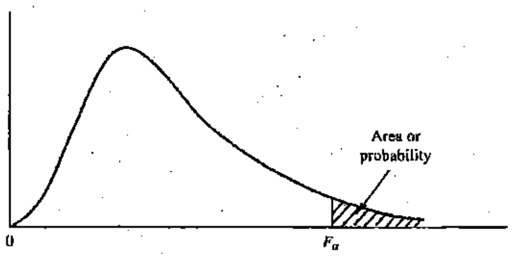
本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷（卡）內作答

參考用

F **TABLE - I**



Entries in the table give F_α values, where α is the area or probability in the upper tail of the *F* distribution. For example, with 12 numerator degress of freedom, 15 denominator degrees of freedom, and a .05 area in the upper tail, $F_{.05} = 2.48$.

Table of $F_{.05}$ Values

Denominator Degrees of Freedom	Numerator Degrees of Freedom																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00