國立中央大學八十七學年度碩士班研究生入學試題卷

所別: 工業管理研究所 甲組 科目: 微積分 共 1 頁 第 1 頁

1. (20%) Find the following limits:

   a) $\lim_{(x,y,z)\to(2,3,1)} \dfrac{y^2-4y+3}{x^2z(y-3)}$ b) $\lim_{(x,y)\to(0,0)} \dfrac{3xy}{5x^4+2y^4}$

   c) $\lim_{x\to 0^+} x \ln x$ d) $\lim_{x\to\infty} \dfrac{x^m}{e^x}$, $m > 0$

2. (20%) Find the areas of the regions bounded by

   a) the graphs of $f(x) = x^2$ and $g(x) = \dfrac{x^2}{2} + 2$.

   b) the graphs of $f(x) = x^2$ and $g(x) = 1 - x^2$.

3. (20%) Find the following limits by l'Hospital's rule.

   a) $\lim_{x\to 0} \dfrac{e^x - 1 - x - x^2/2}{x^2}$ b) $\lim_{x\to 0} \dfrac{e^x - 1 - x - x^2/2}{x^3}$

4. (10%) Use power series to evaluate $\int_0^{0.1} \dfrac{\sin x}{x} dx$ to three decimals. Justify your answer.

5. (10%) Evaluate $\int_0^\infty x^2 e^{-x^2} dx$ from the known integral $\int_0^\infty e^{-x^2} dx = \dfrac{\sqrt{\pi}}{2}$.

6. (10%) Find the equation of the tangent line to the ellipse $x^2 - xy + y^2 = 9$ at (3,0).

7. (10%) Show that $\lim_{n\to\infty} \sqrt[n]{n} = 1$.