國立中央大學九十一學年度碩士班研究生入學試題卷

所別: 工業管理研究所 甲組 科目: 微積分 共 1 頁 第 1 頁

Ten points (10%) each for the following problems.

1. Find the following limits:

(a) $\lim_{x \to 3} \dfrac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}$

(b) $\lim_{x \to +\infty} x \ln\left(\dfrac{x+1}{x-1}\right)$

2. Test the following series for convergence:

(a) $\sum_{n=1}^{\infty} \dfrac{2^n}{n!}$ (4%) (b) $\sum_{k=2}^{\infty} \dfrac{1}{k \ln k}$ (5%) (c) $\sum_{n=1}^{\infty} \left(\dfrac{n}{n+1}\right)^{n^2}$

3. Find the interval of convergence for the power series

$$\sum_{k=0}^{\infty} \frac{(x-2)^k}{2^k \sqrt{k+1}}$$

4. Approximate $\int_0^1 \sin x^3 \, dx$ to four decimal places by use of Taylor series.

5. Evaluate the following integrals:

(a) $\int \ln x \, dx$ (b) $\int_1^4 \dfrac{dx}{\sqrt{x}(\sqrt{x}+1)^3}$ $\dfrac{5}{36}$ (c) $\int_0^{\frac{\pi}{2}} \dfrac{x}{\sqrt{1-x^4}} dx$

6. Compute $\lim_{(x,y,z)\to(0,0,0)} \dfrac{xyz}{x^2+y^2+z^2}$

7. Compute $\lim_{(x,y)\to(0,0)} \dfrac{x^2y^2}{x^4+y^4}$

8. Find a, b, c such that the maximum value of the directional derivative of $f(x,y,z) = axy^2 + byz + cz^2x^3$ at the point $(1,2,-1)$

occurs in the parallel to z-axis (positive direction).

9. Compute the double integral $\iint_S (x-y)^2 \sin^2(x+y) \, dxdy$

Where S = the parallelogram with vertices $(\pi, 0)$, $(2\pi, \pi)$ $(\pi, 2\pi)$, $(0, \pi)$

10. Compute the repeated integral $\int_0^1 \int_{\frac{x}{2}}^{1} e^{y^2} \, dydX$

參考用