所別：工業管理研究所碩士班 乙組 科目：作業研究

1. Consider the following problem.

Maximize $Z = 8x_1 + 4x_2 + 6x_3 + 3x_4 + 9x_5$,

subject to

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 3x_4 &\le 180 \quad \text{(resource 1)} \\ 4x_1 + 3x_2 + 2x_3 + x_4 + x_5 &\le 270 \quad \text{(resource 2)} \\ x_1 + 3x_2 + x_4 + 3x_5 &\le 180 \quad \text{(resource 3)} \end{aligned}$$

and

$$x_j \ge 0, \quad j = 1, \ldots, 5.$$

You are given the facts that the basic variables in the optimal solution are x_3, x_1, and x_5 and that

$$\begin{bmatrix} 3 & 1 & 0 \\ 2 & 4 & 1 \\ 0 & 1 & 3 \end{bmatrix}^{-1} = \frac{1}{27}\begin{bmatrix} 11 & -3 & 1 \\ -6 & 9 & -3 \\ 2 & -3 & 10 \end{bmatrix}.$$

(a) Use the given information to identify the optimal solution. (6分)
(b) Use the given information to identify the shadow prices for the three resources. (6分)

2. Consider the following problem.

Maximize $Z = 2x_1 - 4x_2$,

subject to

$$x_1 - x_2 \le 1$$

and

$$x_1 \ge 0, \quad x_2 \ge 0.$$

(a) Construct the dual problem, and then find its optimal solution by inspection. (4分)
(b) Use the complementary slackness property and the optimal solution for the dual problem to find the optimal solution for the primal problem. (4分)
(c) Suppose that c_1, the coefficient of x_1 in the primal objective function, actually can have any value in the model. For what values of c_1 does the dual problem have no feasible solutions? (4分)

3. You are given the following information about a project consisting of six activities:

Activity	Immediate Predecessors	Estimated Duration
A	—	5 months
B	—	1 month
C	B	2 months
D	A, C	4 months
E	A	6 months
F	D, E	3 months

(a) Construct the project network for this project. (4分)
(b) Find the earliest times, latest times, and slack for each activity. Which of the paths is a critical path? (6分)
(c) If all other activities take the estimated amount of time, what is the maximum duration of activity D without delaying the completion of the project? (4分)

注意：背面有試題

4. A company will be producing the same new product at two different factories, and then the product must be shipped to two warehouses. Factory 1 can send an unlimited amount by rail to warehouse 1 only, whereas factory 2 can send an unlimited amount by rail to warehouse 2 only. However, independent truckers can be used to ship up to 50 units from each factory to a distribution center, from which up to 50 units can be shipped to each warehouse. The shipping cost per unit for each alternative is shown in the following table, along with the amounts to be produced at the factories and the amounts needed at the warehouses.

From \ To	Unit Shipping Cost			
	Distribution Center	Warehouse		
		1	2	Output
Factory 1	3	7	—	80
Factory 2	4	—	9	70
Distribution center		2	4	
Allocation		60	90	

(a) Formulate the network representation of this problem as a minimum cost flow problem. (6分)

(b) Formulate the linear programming model for this problem. (6分)

5. Consider a knapsack problem with n type of items. Assume that n type of items have been ordered, so that

$$\frac{b_1}{w_1} > \frac{b_2}{w_2} > \cdots > \frac{b_n}{w_n},$$

where b_i is the benefit of the i-th item and w_i is the weight of the i-th item. Suppose the knapsack can hold w pounds and there are unlimited number of items for each type. The objective here is to maximize the total benefits.

(a) Consider the same knapsack problem except that the items of type 1 are not going to be used. Formulation this problem as an IP problem and prove the value of objective function for this problem is at most $\frac{b_2 w}{w_2}$. (10 points)

(b) Let's define

$$\tilde{w} = \frac{b_1}{\left(\frac{b_1}{w_1} - \frac{b_1}{w_1}\right)}.$$

Show that if the knapsack can hold w pounds, with $w \geq \tilde{w}$. Then the optimal solution to the knapsack problem with n type of items must use at least one item of type 1. (Hint: Use the result from (a)) (15 points)

(c) It can be shown that the optimal solution to the knapsack problem with n type of items will include at least $1 + \left\lfloor (w - \tilde{w}) / w_1 \right\rfloor$ items of type 1. Suppose we have a w-pound knapsack, where $w = 4000$, to be filled with items listed in the table below.

Item	Weight	Benefit
1	4	11
2	5	12
3	3	7

Combine the above result and use dynamic programming to solve this problem. (15 points)

6. Consider the two-state Markov chain with one-step transition matrix

$$\mathbf{P} = \begin{bmatrix} c & 1-c \\ 1-d & d \end{bmatrix}.$$

Determine values for c and d so that, in steady state, the probability of being in state 1 is p, and the probability of changing state in any one-step transition is a. (10 points)