所別：工業管理研究所碩士班 甲組 科目：微積分

1. Find the Taylor polynomials (of the indicated degree, and at the indicated point) for the following functions:

   (a) (5 points) $f(x)=e^{e^x}$; degree 3, at 0

   (b) (5 points) sin; degree $2n$, at $\frac{\pi}{2}$

   (c) (5 points) exp; degree n, at 1

2. Evaluate each of the following:

   (a) (5 points) $\lim\limits_{n\to\infty}\frac{\sqrt[n]{e}+\sqrt[n]{e^2}+\cdots+\sqrt[n]{e^n}}{n}$

   (b) (5 points) $\lim\limits_{n\to\infty}\left(\frac{1}{n+1}+\cdots+\frac{1}{2n}\right)$

3. (10 points-PROOF) Suppose that $0\le a_n\le b_n$ for all n and $\sum_{n=1}^{\infty}b_n$ converges, show that $\sum_{n=1}^{\infty}a_n$ converges.

4. (15 points-PROOF) Suppose that $\{f_n\}$ is a sequence of functions which are continuous on [a, b], and that $\{f_n\}$ converges uniformly on [a, b] to f. Show that f is also continuous on [a, b].

5. (10 points-PROOF) A real-valued function f defined in (a,b) is said to be convex

   $$f(\lambda x+(1-\lambda)y)\le\lambda f(x)+(1-\lambda)f(y),$$

   for $a<x<b$ $a<y<b$ $0<\lambda<1$. Show that every convex function is continuous.

6. (a) (10 points-PROOF) Let $p>1$ with $1/p+1/q=1$. For a, b, $t>0$, show that

   $$ab\le\frac{a^pt^p}{p}+\frac{a^qt^{-q}}{q}$$

   and ab is the minimum value of the right side.

   (b) (15 points-PROOF) For a_k, $b_k\ge 0$, $p>1$, $1/p+1/q=1$, show that

   $$\sum_1^n a_kb_k\le\left(\sum_1^n a_k^p\right)^{1/p}+\left(\sum_1^n b_k^q\right)^{1/q}$$

   (c) (15 points-PROOF) For a_k, $b_k\ge 0$, $p>1$, show that

   $$\left(\sum_1^n (a_k+b_k)^p\right)^{1/p}\le\left(\sum_1^n a_k^p\right)^{1/p}+\left(\sum_1^n b_k^p\right)^{1/p}$$