國立中央大學109學年度碩士班考試入學試題

所別： 經濟學系 碩士班 不分組(一般生) 共4頁 第1頁

科目： 統計學

本科考試禁用計算器 ＊計算題需計算過程，無計算過程者不予計分 ＊請在答案卷(卡)內作答

1. (15 points) Consider a job training program where Y_i denotes the weekly earning of individual i; D_i is a dummy variable for training status, equal to 1 if i receives the training program and 0 if not; Y_{1i} denotes the potential earning of i provided she/he were to receive the training, whereas Y_{0i} is the potential earning of i if she/he were not.

   We can formally write the potential earning of i by $Y_i = (1 - D_i)Y_{0i} + D_i Y_{1i}$. It is of our interest to identify the average treatment effect on the treated (ATT) from the training program, that is, $E(Y_{1i} - Y_{0i}|D_i = 1)$.

   (a) (3 points) Explain the difference between $E(Y_{1i} - Y_{0i}|D_i = 1)$ and $E(Y_{1i} - Y_{0i})$.

   (b) (3 points) Propose a sample version point estimate for $E(Y_{1i}|D_i = 1) - E(Y_{0i}|D_i = 0)$.

   (c) (6 points) Show that $E(Y_{1i}|D_i = 1) - E(Y_{0i}|D_i = 0) \neq E(Y_{1i} - Y_{0i}|D_i = 1)$.

   (d) (3 points) Give sufficient condition(s) such that $E(Y_{1i}|D_i = 1) - E(Y_{0i}|D_i = 0) = E(Y_{1i} - Y_{0i}|D_i = 1)$.

2. (5 points) Let W be a random variable whose moment generating function is:

   $$M_W(t) = \left(\frac{1}{10} + \frac{2}{5}e^{t} + \frac{2}{15}e^{-t} + \frac{1}{15}e^{4t} + \frac{1}{10}e^{-7t} + \frac{3}{15}e^{20t}\right).$$

   Find $Prob(W \leq 3.14159)$.

參考用

3. (10 points) Consider two random variables X and Y as follows.

   (a) (5 points) Suppose X and Y are independent and distributed by Poisson(λ_1) and Poisson(λ_2), respectibely. Find the pdf of $X + Y$. (hint: a random variable X~Poisson(λ) if and only if $f_X(x) = \exp(-\lambda)\,\lambda^x/x!$)

   (b) (5 points) Suppose X and Y are independent and distributed by the exponential distributions Exp(β_1) and Exp(β_2), respectibely. Find the pdf of $X + Y$. (hint: a random variable X~Exp(λ) if and only if $f_X(x) = \lambda\exp(-\lambda x)$)

注意：背面有試題

所別： 經濟學系 碩士班 不分組(一般生) 共4頁 第2頁

科目： 統計學

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4. (10 points) Suppose $X \sim \text{Poisson}(\lambda)$. Consider the following hypotheses.

   H_0: $\lambda = 3.4$ vs H_A: $\lambda = 1.5$

   Now consider a test that rejects the null hypothesis when $X \leq 2$ is observed. Find the size and the power of this test.

5. (25 points) A random sample ($x_1, x_2, x_3, \ldots x_{25}$) is drawn from a population $X \sim N(\mu, 36)$. Let $\bar{X}$ and s represent the mean value and standard deviation of the random sample, respectively. ($\alpha = 0.05$)

   (a) (10 points) How is x_1 distributed? Expected value=? Standard deviation=?

   $Cov((x_1, x_2)$=? $Var((x_1 + x_2)/2)$=?

   (b) (15 points) Identify mistakes involved in the following hypothesis test:

   H_0: $\bar{X}=10$

   H_1: $\bar{X} \neq 10$

   Reject H_0 if *p-value* < 0.05, where *p-value*=Prob($t \geq \frac{\bar{X}-10}{s/\sqrt{25}}$); or reject H_0 if 10 is not in the 95% confidence interval estimate of the population mean ($=\bar{X} \pm t_{.025}(24) \times \frac{s}{\sqrt{25}}$).

6. (25 points) Two random samples of equal size are drawn from firms in Region A and B in Taiwan, $n_1 = n_2 = 23$. Suppose firms in the two regions follow the Cobb-Douglas production technology with two inputs factors, labor (L) and capital (K). If the model is regressed individually for the two regions. The sums of squared residuals for Region A and B are 340 and 220, respectively. If the firm data from the two regions are pooled together and only one single equation is regressed, the sum of squared residuals is 728. Is it appropriate to pool all the data together and do the empirical study? Conduct a hypothesis test.

   (a) (5 points) State the corresponding null and alternative hypotheses for the test.

   (b) (5 points) What is the test statistic used in your hypothesis test? How is the test statistic distributed?

   (c) (5 points) Can you conclude that firms in the two regions have exactly the same production technology?

   (d) (5 points) Suppose the regression model is estimated separately for the two regions.

參考用

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How would you test whether the production technology for firms in Region A is constant returns to scale? What test statistic would you use? What is the decision rule in your test?

(e) (5 points) Following question (d), how would you test if the output elasticity of labor for firms in Region A is larger than that in Region B? What test statistic would you use? What is the decision rule in your test?

7. (10 points) Consider the following regression model $Y_i = \beta D_i + u$, where D_i is an endogenous dummy variable. In order to retrieve the identification, we have an instrumental variable z_i available for estimation. Assume that z_i is also binary. Prove that β_{IV}, the IV estimator of β, can be shown as

$$\frac{E(Y_i|z_i = 1) - E(Y_i|z_i = 0)}{Prob(D_i|z_i = 1) - Prob(D_i|z_i = 0)}$$

Appendix I

Table of Poisson Probabilities

	λ				
x	1.1	1.2	1.3	1.4	1.5
0	0.3329	0.3012	0.2725	0.2466	0.2231
1	0.3662	0.3614	0.3543	0.3452	0.3347
2	0.2014	0.2169	0.2303	0.2417	0.251
3	0.0738	0.0867	0.0998	0.1128	0.1255
4	0.0203	0.026	0.0324	0.0395	0.0471
5	0.0045	0.0062	0.0084	0.0111	0.0141
6	0.0008	0.0012	0.0018	0.0026	0.0035
7	0.0001	0.0002	0.0003	0.0005	0.0008
8	0	0	0.0001	0.0001	0.0001
9	0	0	0	0	0

參考用

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	λ				
	3.1	3.2	3.3	3.4	3.5
0	0.045	0.0408	0.0369	0.0344	0.0302
1	0.1397	0.1304	0.1217	0.1135	0.1057
2	0.2165	0.2087	0.2008	0.1929	0.185
3	0.2237	0.2226	0.2209	0.2186	0.2158
4	0.1734	0.1781	0.1823	0.1858	0.1888
5	0.1075	0.114	0.1203	0.1264	0.1322
6	0.0555	0.0608	0.0662	0.0716	0.0771
7	0.0246	0.0278	0.0312	0.0348	0.0385
8	0.0095	0.0111	0.0129	0.0148	0.0169
9	0.0033	0.004	0.0047	0.0056	0.0066
10	0.001	0.0013	0.0016	0.0019	0.0023
11	0.0003	0.0004	0.0005	0.0006	0.0007
12	0.0001	0.0001	0.0001	0.0002	0.0002
13	0	0	0	0	0.0001
14	0	0	0	0	0

參考用

Appendix II

***F* Distribution**

Denominator Degrees of Freedom	Area in Upper Tail	Numerator Degrees of Freedom 1	2	3	4
30	.10	2.88	2.49	2.28	2.14
	.05	4.17	3.32	2.92	2.69
	.025	5.57	4.18	3.59	3.25
40	.10	2.84	2.44	2.23	2.09
	.05	4.08	3.23	2.84	2.61
	.025	5.42	4.05	3.46	3.13
60	.10	2.79	2.39	2.18	2.04
	.05	4.00	3.15	2.76	2.53
	.025	5.29	3.93	3.34	3.01

Entries in the table give $F\alpha$ values, where α is the area or probability in the upper tail of the *F* Distribution. For example, with 1 numerator degrees of freedom, 30 denominator degrees of freedom, and a 0.10 area in the upper tail, $F_{0.10}=2.88$.

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